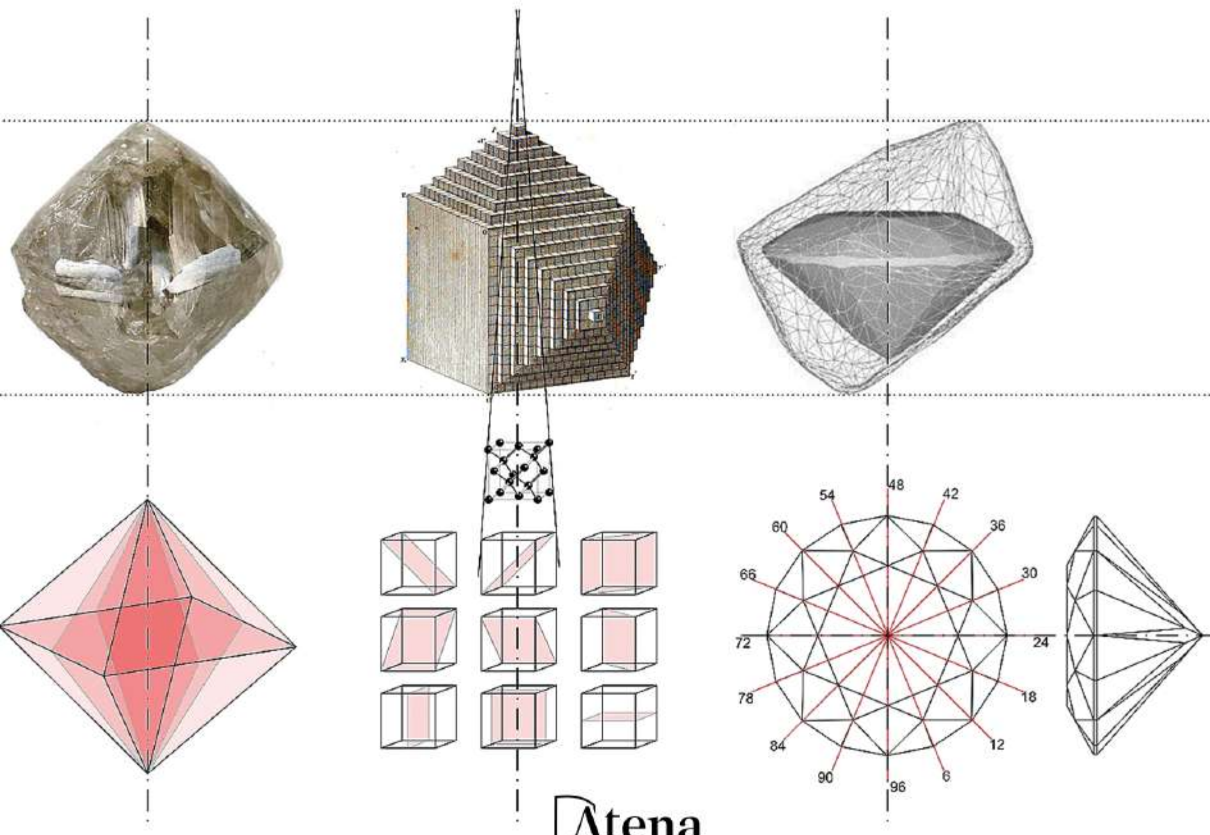
Nicola Pisacane, Pasquale Argenziano, Alessandra Avella

GEMSTONES' DRAWING/DESIGN

Geometry Models Representations

Preface by Paolo Belardi



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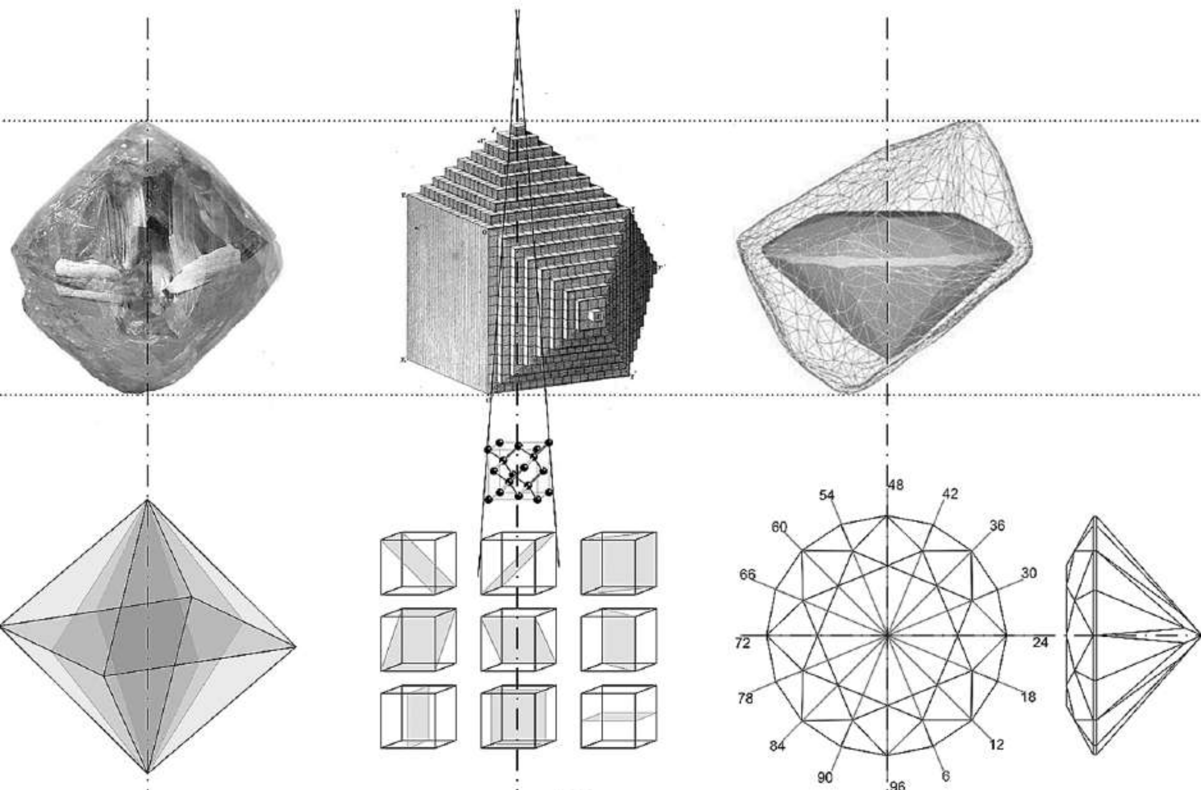

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PREFACE

THE DRAWING'S EYE

by Paolo Belardi

While reading this book, I was reminded of Leon Battista Alberti's winged human eye. An eye that, as is well known, moves quickly, but also freely, between past, present, and future – specifically between analog and digital – precisely because it can recognize the reality of natural facts by overcoming the unreality of artificial fiction. And that, with its absolute disenchantment, is a prelude to Leonardo da Vinci's eye when, in a famous passage of the Codex Arundel 263 (fol. 155 recto), he declares his «bramosa voglia»[1] to «vedere la gran copia delle varie e strane forme fatte dalla artifiziosa natura»[2] which pushes it to explore the depth of a «oscura spilonca»[3] to «vedere se là entro fusse qualche miracolosa cosa.»[4] confessing that mix of «paura e desiderio»[5] in the face of the unknown that circularly re-proposes the rhetorical question – «Quid Tum» – with whom Alberti seals his winged human eye and which also transpires from the book co-authored by Nicola Pisacane, Pasquale Argenziano and Alessandra Avella, which locks Drawing, Geometry and Design in a Borromean knot capable of revealing the humanistic face of the Mineralogical Sciences, prefiguring new ideational potential in the field of Jewellery Design.

But let's go in order.

The book is dedicated to the Drawing/Design of precious stones and has its roots in the educational field, as the authors state. It is divided into three chapters, accompanied by illustrations so eloquent as to return a sort of autonomous compendium.

In the first chapter – Crystals' geometry in mineral and faceted gemstones – Nicola Pisacane goes far beyond the usual citation of the works of the great fifteenth-century treatises (from Piero della Francesca's De quinque corporibus regularibus to Luca Pacioli's Divina Proportione). He focuses his analyses on the later, less known but equally meaningful studies of Jean-Baptiste Romé de L'Isle, Abbot René Just Haüy, and Auguste Bravais, thus emphasizing with originality, as well as lucidity, the centrality of Geometry in the study of crystals and faceted gems.

In the second chapter – Crystallographic forms' models, from tangible to digital – Alessandra Avella explores the concept of the model as a communicative

1 "eager desire" [Ed.]
2 "see the great copy of the various and strange forms made by artificial nature" [Ed.]
3 "dark cave" [Ed.]
4 "see if there was something miraculous in there." [Ed.]
5 "fear and desire" [Ed.]

artifact aimed at translating crystalline forms from the microscopic to the macroscopic scale, starting from the morphological crystallography of Jean-Baptiste Romé de L'Isle: an approach typical of historical pedagogical experiences (from the Pestalozzi method to the Montessori one, passing through the Bauhaus laboratory), but updated in a parametric key by prefiguring the compelling possibility of modeling 'new' crystals.

Finally, in the third chapter – The scientific representation of crystal polyhedra. Insights and methodological innovations (XVII-XX Centuries) – Pasquale Argenziano, highlights the synergistic interrelationships between the Parallel Projections for the study of crystals and faceted gemstones by René Just Haüy with the better-known ones by Gaspard Monge and William Farish, highlighting the theoretical acquisitions of Projective Geometry matured in the early Nineteenth Century in the French polytechnic milieu, and still other synergies with the theorization of crystallographic stereography pursued by Franz Ernst Neumann and William Hallowes Miller, extending the trigonometric foundations of cartography to the microscopic scale.

The book ends in the best possible way. Looking to the future, it opens the doors of research to JIM-Jewel Information Modeling, understood as a field of experimentation capable of combining parametric modeling with block-chain certification.

It could not have been otherwise; Because this book does not limit itself to reporting on a passionate and shared path of study and teaching – which would represent a qualifying fact in itself – but establishes new points of view, confirming the universal role of drawing-thought as a privileged eye for the recognition of "miraculous things"; even in only apparently unusual fields such as Mineralogy, Crystallography, and Gemology.

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INTRODUCTION

This book is the first systematization of our research on the Gemstones' Drawing/ Design – begun four years ago – presented at national and international conferences of the disciplines of Geometry, Drawing and graphic Representation Sciences, and in the International Lectures, and also in the PhD Course at foreign Universities. (Argenziano et al., 2022, 2024; Avella et al., 2024; Pisacane et al., 2021, 2023a, 2023b)

Our research is enhanced by collaborations with Italian institutions that teach crystallography (i. e., the Michelangelo Museum into the *Istituto Tecnico Statale Michelangelo Buonarroti* in Caserta) and international companies that experiment in gemology.

The investigative starting point was first linked to teaching in the Drawing Workshops of the Department of Architecture and Industrial Design of the University of Campania Luigi Vanvitelli, and was subsequently strengthened by the numerous bibliographical and iconographical sources consulted. In the first phase, the study of crystal polyhedra facilitated the students' understanding of abstract concepts of geometry (i.e. symmetry, proportional ratio) and Descriptive Geometry (Parallel and Central projections) through case studies that are close to tangible experience. In the second phase, the variety and quantity of textual and iconographic sources (including von Linné, Romé de L'Isle, Haüy, Miller, and Bravais), consulted to organize the lectures mentioned above, have revealed to us a 'new' cultural scenario that is the foundation of Drawing because many theoretical concepts (irregular polyhedra, symmetries of polyhedra, micro-macro model) and many graphic applications (Axonometries and Orthogonal Projections) have had their definition widely shared in the Twentieth Century, also thanks to the studies of Earth Sciences carried out between the eighteenth and nineteenth centuries.

In the following pages, we propose the Gemstones' Drawing/Design, and by geometric extension that of mineral crystals, from three different points of view than the more well-known ones, such as the design of the jewel, and the iconography of the jewel worn in historical portraiture, to which we referred at the beginning, thanks to the fundamental pages on the Drawing by Prof. Gaspare De Fiore. (De Fiore, 1983)

The three chapters of this book focus precisely on our three points of view that we share, like every phase of this research.

In the first chapter, the theory of crystals is treated concerning the geometrical principles that govern both the atomic structure of minerals and the spatial configurations of gems obtained through faceting processes starting from the raw ore. The theoretical principles of polyhedron geometry govern the morphology of crystals both at the microscopic scale and at the scale of a valuable accessory.

In the second chapter, the geometry of crystallographic forms and their modeling is deepened from theoretical abstraction to the tangible concreteness of analog and digital models. The study compares the crystallographic forms' models on display at the

Michelangelo Museum in Caserta, the homologous iconography of the main Italian and foreign catalogs of educational material, and the most recent dedicated scientific modeling software.

In the third chapter, the graphic representation of crystal polyhedra is examined through the experiments of the main mineralogists from the Seventeenth to the early Twentieth Century. The problem of the dual scientific representation of solids (macroscopic and microscopic ones, irregular and regular ones) on the plane has been tackled and solved in the French and German polytechnic environments, by different methodological and technical approaches. Those little-known scientific discoveries stand alongside the other well-known ones of the fathers of the Science of Representation.

An extensive iconographic apparatus illustrates the chapters. Some images have been selected from the pages of the main books on mineralogy, geometrical crystallography, and gemology, published between the Sixteenth and Nineteenth centuries. Other images are drawings and diagrams that we have developed to support the research, or that our students have developed during the Drawing Workshops.

CRYSTALS' GEOMETRY IN MINERAL AND FACETED GEMSTONES

by Nicola Pisacane

CRYSTALLOGRAPHIC FORMS. 3D CONFIGURATION AND THEIR GEOMETRIC PROPERTIES

Crystallographic studies recognize the role played by geometry in the mineralogical assessment of crystal structures, introducing a branch called "Geometric crystallography" that analyzes a mineral's geometric and morphologic structure. Instead, Physical and chemical crystallography study different topics related to minerals and gemstones. (Sansoni, 1892; Engel, 1986)

Assuming crystal forms are polyhedra, it is interesting to note how crystallography refers to the principles of 3D geometry in studying and classifying these forms. The study of regular polyhedra originates in Plato's Timeo, in which the configurative properties and their symbolic value concerning the fundamental elements of the Earth are described. Over time, especially since the Renaissance, the renewed interest in geometry led scientists to study regular shapes again. Suffice it to think of the contributions of the treatises of Leon Battista Alberti, Piero della Francesca, and Fra Luca Pacioli (Sgrosso, 2001; Vagnetti, 1979), which were followed by the studies of Kepler for the possible applications in the astronomical field, and those of Descartes and Euler who were the first to formulate considerations on the relations between vertices, edges and faces that led to the algebraic relation that bears the name of Euler himself and which found application in geometric crystallography itself (Sansoni, 1892, pp. XI–XII). In crystallography, however, it was only in the second half of the 18th century that geometric solids representing crystal forms were characterized as polyhedra and assumed all their properties.

Starting from the studies by Jean-Baptiste Romé de L'Isle (1736-1790) (Romé de L'Isle, 1783) followed by the ones from Abbot René Just Haüy (1743-1822) (Haüy, 1784, 1817, 1822) crystallography takes on a scientific character and in the following decades research and theories will follow transferring principles of spherical trigonometry and geometry of polyhedrons to crystallography, determining a new approach to the study of minerals (Borchardt-Ott, 2011).

The geometric organization of crystals was introduced during a period in which the observation of mineral species led to a scientific approach to studying the crystallographic, physical, and chemical properties of minerals. In this period, starting from the work of Haüy, the classification of crystal forms and symmetries began to be systematized, represented in his treatise using drawn tables **(Figg. 1, 2, 3)**.

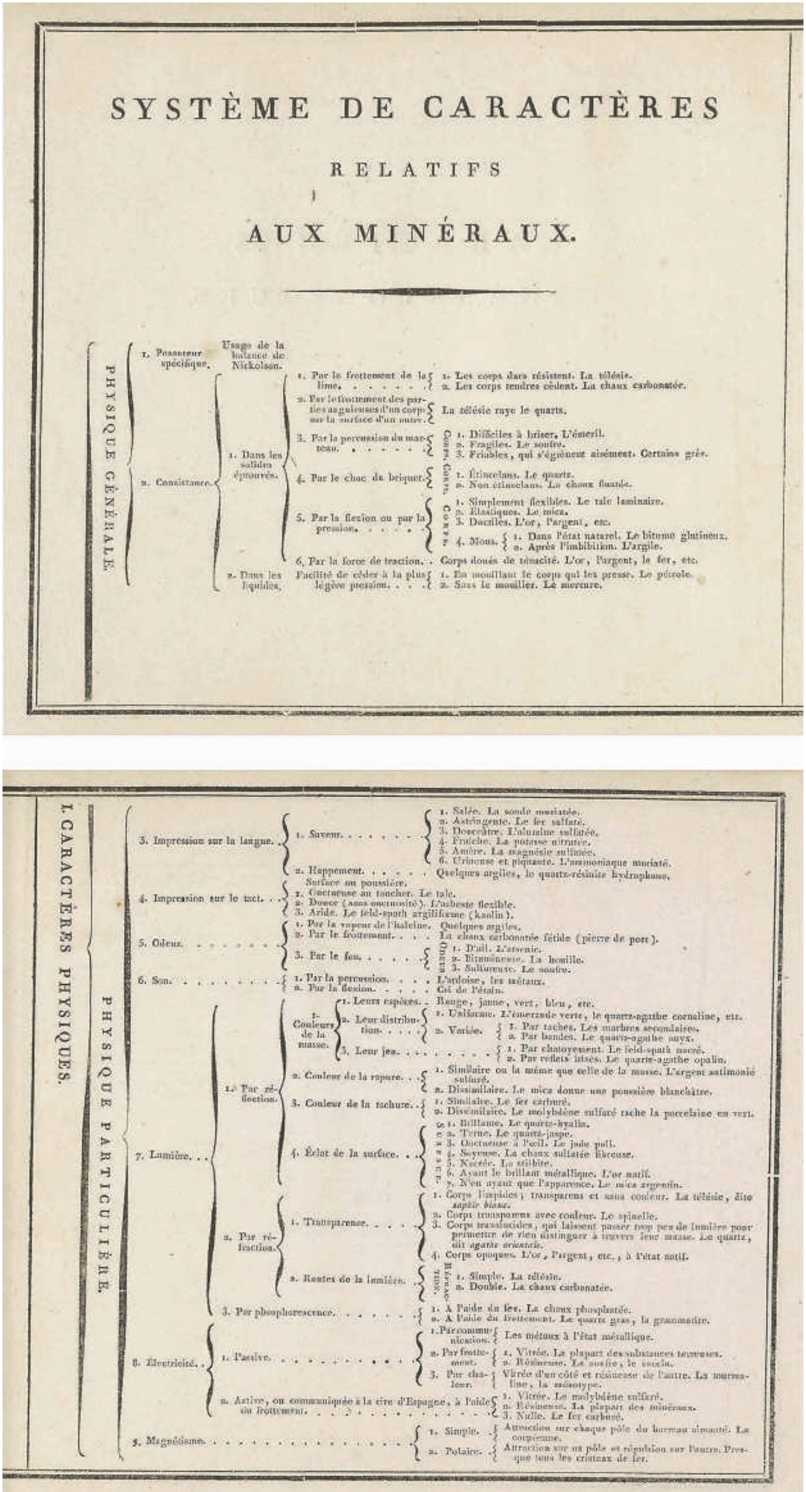
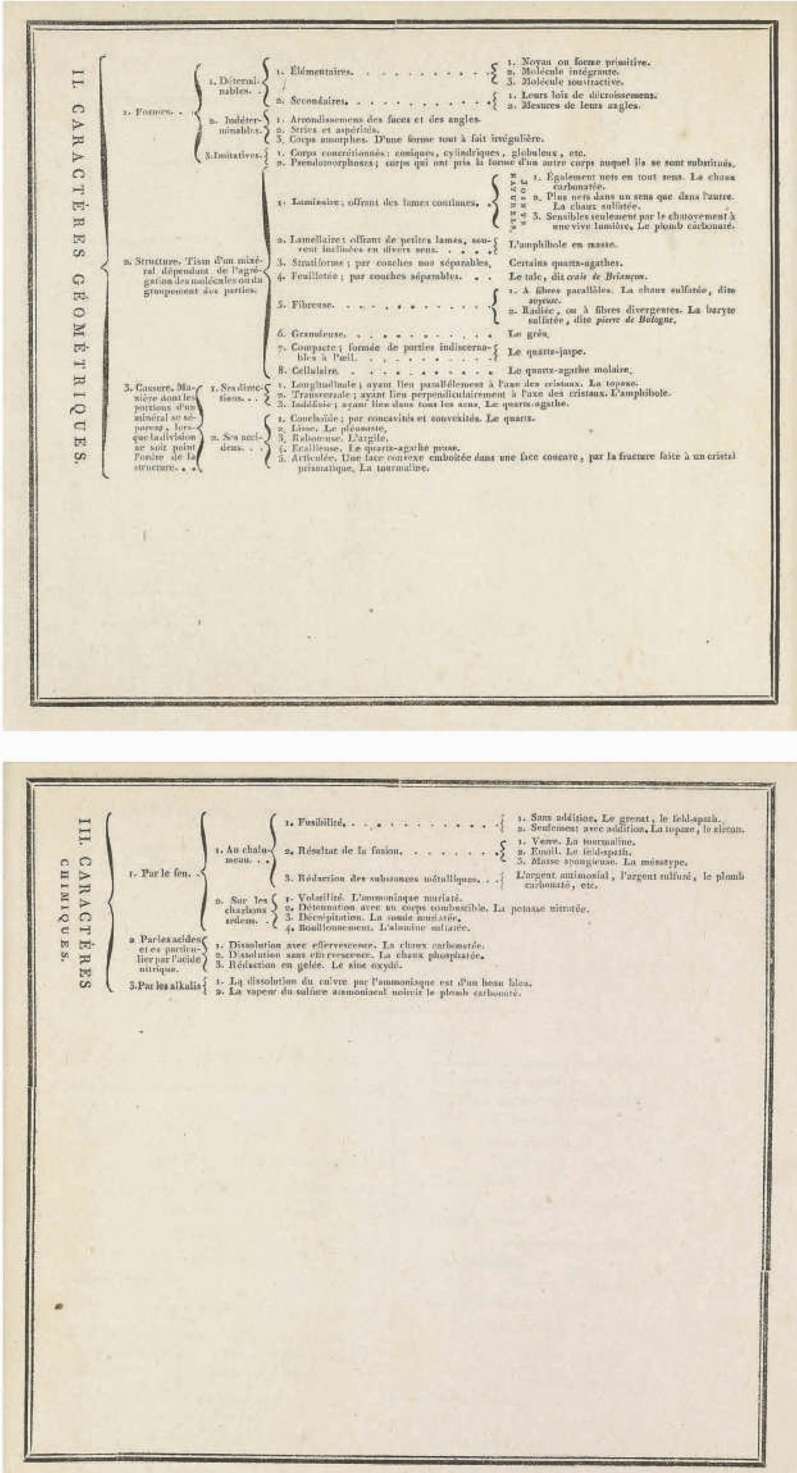


Fig. 1. Table of physical characters of minerals. (Haüy, 1801)



II. CARACTÈRES GÉOMÉTRIQUES.

- 1. Formes.
  - 1. Déterminables.
    - 1. Élémentaires.
      - 1. Noyau ou forme primitive.
      - 2. Molécule intégrante.
      - 3. Molécule soustractive.
    - 2. Secondaires.
      - 1. Leurs lois de décroissement.
      - 2. Mesures de leurs angles.
  - 2. Indéterminables.
    - 1. Arrondissemens des faces et des angles.
    - 2. Stries et aspérités.
    - 3. Corps amorphes. D'une forme tout à fait irrégulière.
  - 3. Imitatives.
    - 1. Corps concrétionnés; coniques, cylindriques, globuleux, etc.
    - 2. Pseudomorphoses; corps qui ont pris la forme d'un autre corps auquel ils se sont substitués.
- 2. Structure. Tissu d'un minéral dépendant de l'agrégation des molécules ou du groupement des parties.
  - 1. Laminaire; offrant des lames continues.
    - Joints naturels.
      - 1. Également nets en tout sens. La chaux carbonatée.
      - 2. Plus nets dans un sens que dans l'autre. La chaux sulfatée.
      - 3. Sensibles seulement par le chatoyement à une vive lumière. Le plomb carbonaté.
  - 2. Lamellaire; offrant de petites lames, souvent inclinées en divers sens. — L'amphibole en masse.
  - 3. Straiforme; par couches non séparables. — Certains quartz-agathes.
  - 4. Feuilletée; par couches séparables. — Le talc, dit *craie de Briançon*.
  - 5. Fibreuse.
    - 1. À fibres parallèles. La chaux sulfatée, dite soyeuse.
    - 2. Radiée, ou à fibres divergentes. La baryte sulfatée, dite *pierre de Bologne*.
  - 6. Granuleuse. — Le grès.
  - 7. Compacte; formée de parties indiscernables à l'œil. — Le quartz-jaspe.
  - 8. Cellulaire. — Le quartz-agathe molaire.
- 3. Cassure. Manière dont les portions d'un minéral se séparent, lorsque la division ne suit point l'ordre de la structure.
  - 1. Ses directions.
    - 1. Longitudinale; ayant lieu parallèlement à l'axe des cristaux. La topaze.
    - 2. Transversale; ayant lieu perpendiculairement à l'axe des cristaux. L'amphibole.
    - 3. Indéfinie; ayant lieu dans tous les sens. Le quartz-agathe.
  - 2. Ses accidens.
    - 1. Conchoïde; par concavités et convexités. Le quartz.
    - 2. Lisse. Le pléonaste.
    - 3. Raboteuse. L'argile.
    - 4. Écailleuse. Le quartz-agathe pyrome.
    - 5. Articulée. Une face convexe emboîtée dans une face concave, par la fracture faite à un cristal prismatique. La tourmaline.

III. CARACTÈRES CHIMIQUES.

- 1. Par le feu.
  - 1. Au chalumeau.
    - 1. Fusibilité.
      - 1. Sans addition. Le grenat, le feld-spath.
      - 2. Seulement avec addition. La topaze, le zircon.
    - 2. Résultat de la fusion.
      - 1. Verre. La tourmaline.
      - 2. Émail. Le feld-spath.
      - 3. Masse spongieuse. La mésotype.
    - 3. Réduction des substances métalliques. — L'argent antimonial, l'argent sulfuré, le plomb carbonaté, etc.
  - 2. Sur les charbons ardens.
    - 1. Volatilité. L'ammoniaque muriaté.
    - 2. Détonnation avec un corps combustible. La potasse nitratée.
    - 3. Décrépitation. La soude muriatée.
    - 4. Bouillonnement. L'alumine sulfatée.
- 2. Par les acides et en particulier par l'acide nitrique.
  - 1. Dissolution avec effervescence. La chaux carbonatée.
  - 2. Dissolution sans effervescence. La chaux phosphatée.
  - 3. Réduction en gelée. Le zinc oxydé.
- 3. Par les alkalis.
  - 1. La dissolution du cuivre par l'ammoniaque est d'un beau bleu.
  - 2. La vapeur du sulfure ammoniacal noircit le plomb carbonaté.

Fig. 2. Table of geometrical and chemical characters of minerals. (Haüy, 1801)

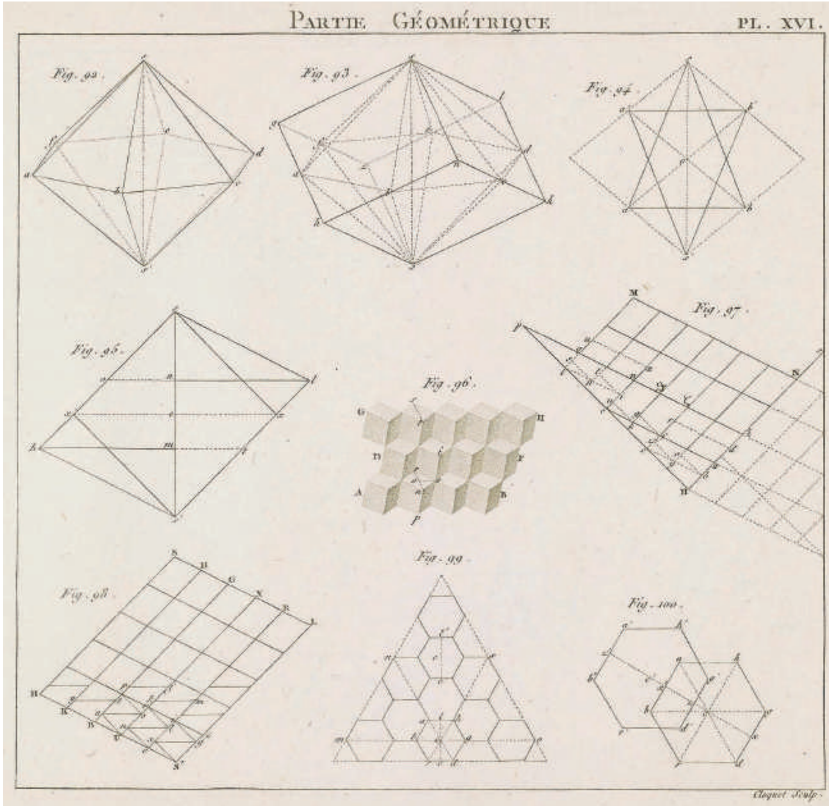


Fig. 3. Geometric part, planche XVI. (Haüy, 1801)

This approach finds its foundation in the study of the arrangement in 3D space of atoms according to ordered, regular, constant, and repetitive series, following the geometric rules of tessellation of space. Crystallographic systems and their geometric lattices are classified on this spatial organization.

In particular, the two main different crystallographic models are based on various geometric approaches. The first one was introduced by Haüy and is based on an aggregation of solid polyhedrons; the second one was defined by the French physicist Auguste Bravais (1811-1863), who set a classification method based on punctual positions of the crystal atoms (Bravais, 1849, 1866).

Both geometric models (solid and point) originate from the physical and atomic structure. The relationship between dimension and shape defines the spatial organization of the elementary cell, which, by reiterating in space, gives life to the crystal and its 3D configuration, which can also differ from the geometric form of the constituent cell. The declination of the different crystallographic systems is based on the 3D spatial organization of axis, edges, and vertices that define the classification of crystal forms according to groups, systems, and classes strictly related to the degrees of symmetry (binary, ternary, quaternary, senary). The premise of crystal classification is to assume crystals are polyhedral. Given the number and variability of crystals concerning chemical composition and atomic structure, 32 classes of symmetry have been identified, schematized in 7 crystallographic systems (cubic, trigonal, tetragonal, hexagonal, orthorhombic, monoclinic and triclinic), grouped in turn into three crystallographic groups (monometric, dimetric, trimetric). **(Fig. 4)**

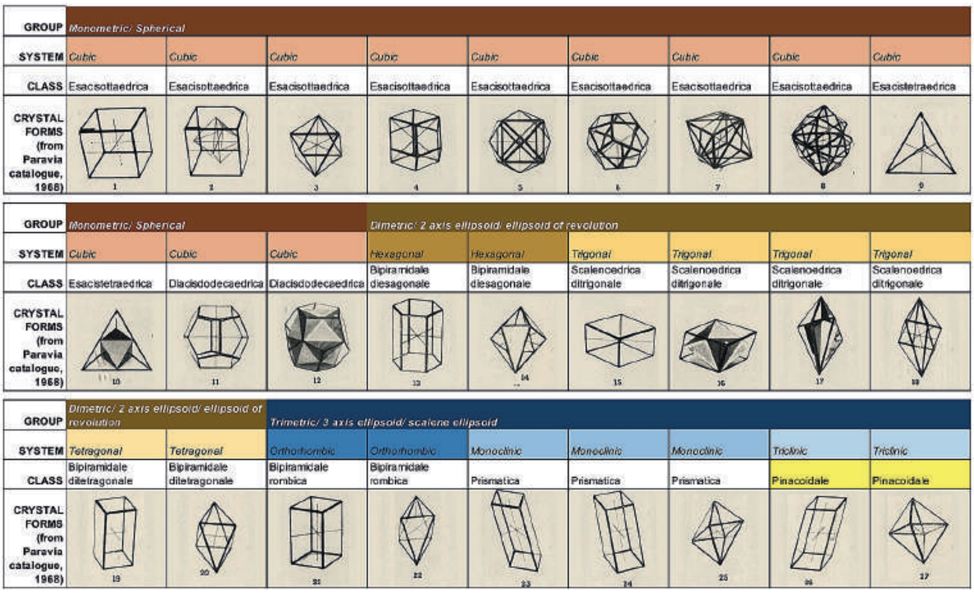


Fig. 4. Classification of crystal forms according group, system and class based on the glass models shown in (Paravia & C., 1968); Diagram by P. Argenziano, A. Avella, and N. Pisacane, 2024.

Specifically, crystallographic groups are organized according to the spatial arrangement of the crystallographic axes that find their common origin in the center of the crystal and, from this, are oriented according to well-defined angles and directions. Already Haüy, in 1782, stated that the reference axes of a crystal are three oriented lines parallel to three converging and non-coplanar edges. The main classification of crystals on the axes is based on recognizing three different crystal groups. The monometric group's parameters are the same, and the crystal axis has the exact dimensions. The group is also named spherical group according to the equal length of axes that can be assumed as the diameter of a sphere. In the dimetric group, a direction is prevalent in the other two. The group is also based on an ellipsoid of revolution obtained by the rotation of an ellipse around the major axis. In the trimetric group, all parameters are different. The group is also defined as a scalene ellipsoid according to the three different lengths of the axis.

The organization of the seven possible crystallographic systems instead is based on the length of the faces and the angular ratio between them and, therefore, on the type of symmetry created between them. Depending on the length of the axes and the angles between them the cubic system (axes of equal size and three right angles), tetragonal (two equal sides and one unequal but three equal and right angles), orthorhombic (three unequal sides and three right angles), trigonal (three equal sides to each other and three angles equal to each other but differently inclined concerning the right angle), hexagonal (two equal sides that form an angle of 120° between them, while the other two angles are straight and the third side of different length than the previous ones), monoclinic (three unequal sides and only two right angles) and triclinic (three sides and three unequal angles) are determined. **(Fig. 5)**

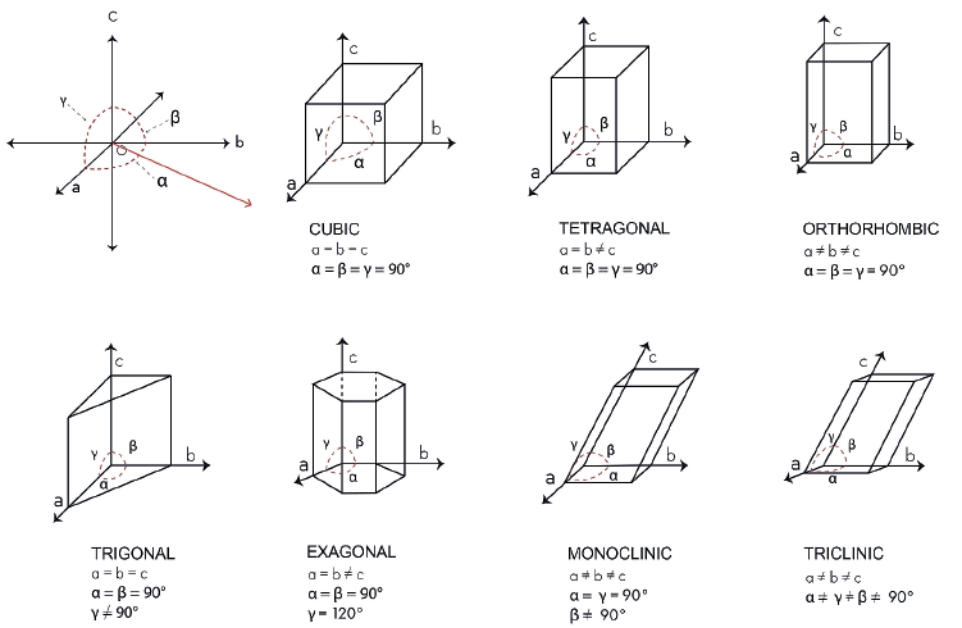


Fig. 5. Crystal axes system (top left): 'a' (crystal length), 'b' (crystal width) and 'c' (crystal height), the origin 'O' and the angles between axes 'α', 'β' and 'γ'. The crystallographic systems are defined by the angles and lengths between the parts; Drawing by Francesca Fabozzi, 2022.

These arrangements in 3D space, in addition to the seven forms mentioned above, also identify different symmetry ratios between the geometrically and physically homologous portions of the crystal (Mottana et al., 1977). Also, in the case of symmetries, three possible types are defined: symmetry to a plane or reflection about its surfaces, according to an axis or rotation about it, and according to a point or inversion about the same point. The symmetry of a crystal structure, however, should not be understood and referred only to the external geometric shape of the crystal but also to that recognizable after a chemical-physical analysis of the crystal to allow it to trace its minimum geometric structure. This symmetry - called real or true symmetry - can be classified according to 32 classes, each corresponding to one or more typical mineralogic species. Each class also corresponds to a crystallographic system based on geometric shapes based on the combination of several regular polyhedrons that define the properties of individual minerals and their predisposition to be cut according to particular shapes or geometries.

Discussing crystal groups can open to interesting considerations of geometric properties that characterize each one.

Geometric characteristics of monometric group crystals can be classified according to the following:

- Polygonal faces of the crystal that are all similar in shape and dimension;
- Axes are spatially located according to the condition $\alpha=\beta=\gamma=90°$; $a=b=c$, with α, β, γ angles between axes and a, b, c as lengths of the axes themselves;

This condition can be identified only in the cubic system. This group includes the crystal forms of the octahedron, the pentagon-dodecahedron, the triakis-octahedron, and the icositetrahedron.

The following geometric conditions characterize the systems of the dimetric group crystals.

For the hexagonal system:

- Axes are especially located according to the condition $\alpha=\beta=90°$; $\gamma=120°$ $a=bc$, with α, β, γ angles between axes and a, b, and c as lengths of the axes themselves. This system is also characterized by four axes, three of which are on the same plane.

For the tetragonal system:

- Axes are located according to the condition $\alpha=\beta=\gamma=90°$; $a=bc$, with α, β, γ angles between axes and a, b, and c as the axes' lengths.

The hexagonal group includes the crystal forms of the rhombohedron, the hexagonal bipyramid, the hexagonal prism, and the scalenohedron. The tetragonal system consists of the tetragonal bipyramid. These forms belonging to the dimetric group have a principal axis that defines the main direction of the crystal form. This axis always intersects the barycenter of polygons on parallel planes or vertices opposite the crystal center. On the other hand, the minor axes cross to the midpoints of some edges of the crystal forms or opposite vertices and are always orthogonal to the principal axis. This axis also defines the direction of the rotation axis of the ellipsoid, identifying the group. In contrast, the other axes identify the center and plane of the maximum circle of the same ellipsoid.

The following geometric conditions characterize the trimetric group crystals.

For the orthorhombic system:

- Axes angles are $\alpha = \beta = \gamma = 90°$;

For the monoclinc system:

- $\alpha = \gamma= 90°$, $\beta \neq 90°$;

For the triclinic system:

- since the axes are all oblique to each other they are spatially located according to the condition $\alpha \neq \beta \neq \gamma \neq 90°$ (Mottana et al., 1977).

This group includes the rhombic prism and rhombus (rhombic bipyramid) models for the orthorhombic system, the monoclinic prism model for the monoclinic system, and the triclinic octahedron for the triclinic system.

Additional geometric properties in crystal form can be found in hemihedral and symmetry, which are rules that define specific mineral characteristics.

In crystallography, hemihedral forms link some crystal shapes from others by halving the number of faces. Within the crystal forms of the monometric group, for example, the pentagon-dodecahedron, characterized by 12 faces, finds its hemihedral form in the tetrakis-hexahedron, which has 24 faces.

Some hemihedral forms exemplifying the dimetric group are the rhombohedron (6 faces), a double pyramid with a hexagonal base (12 faces), classifiable in the hexagonal hemihedral, and the scalenohedron (12 faces), a double pyramid with a dodecagonal base (24 faces) in the rhombohedral hemihedral (Sansoni, 1892). In the case of the rhombohedron, some triangular faces of the hexagonal bipyramid are all coincident with the crystal faces.

Symmetry constraints in crystallography instead determine other forms of classification. In general, symmetry conditions can be classified according to different operators - planes, axes, and centers - or their combinations. The geometrical configuration of monometric group crystals defines the centers of symmetry coincident with the origin of the crystallographic axes, symmetry axes, and symmetry planes (3 or 9) (Sansoni, 1892). Different conditions of symmetry are determined by the position of the center of symmetry, defined as the midpoint of the straight line that crosses the solid surface, and the axis of symmetry as the segment around whom rotation of part of the external surface of the form after a rotation of 360°/n determine the perfect overlapping and the symmetry plan as a reflection plan. Classification of crystal forms is also arranged according to the divisor of the round angle and is identified in four possible cases: 2, 3, 4, and 6.

In addition, optical phenomena related to light propagation within a mineral are strictly connected to geometrical characteristics. Concerning this, a high level of symmetry identified in the monometric group ensures a higher performance of the light reflection in the mineral than in the trimetric group due to the reduced symmetry conditions. So, light reflection is inversely proportional to the anisotropy properties of the precious material, according to whom the different refractive index values depend on the atomic aggregation lattice. In general, the light reflection behavior is defined by surfaces called optical indicators that determine the crystal's refractive indices using a 3D crystal form model.

After the development of the classification of minerals, according to Haüy, the studies in mineralogy led to the transition from solid to punctual geometric models for the classification of crystal structures. Specifically, it is due to Auguste Bravais the systematization in 1848 of the models of geometric arrangement of the molecular entities of the structure of crystals. Crystallography following Haüy principles identified seven possible crystallographic systems; subsequently, Bravais refined this classification through the introduction of spatial lattices and fourteen possible models of 3D organization based on the arrangement in space of the particles that make up the crystal itself according to sets of points through infinitely repeatable patterns in space. In his treatise, he organizes the classification of crystal forms according to their polyhedral shape, the symmetry class, and the minimum number of vertices (Bravais, 1866). A large number of drawn tables are also added to the text, thus confirming the role of drawing and illustration in this field of science. **(Figg. 6, 7)**

POLYÈDRE			SYMBOLE DE LA SYMÉTRIE DU POLYÈDRE.	CLASSE du POLYÈDRE.	NOMBRE MINIMUM DES SOMMETS DE			
					1re ESPÈCE.	2me ESPÈCE.	3me ESPÈCE.	4me ESPÈCE.
asymétrique.			oL, oC, oP	1re	1	1	1	1
symétrique	dépourvu d'axes.		oL, C, oP	2me	2	2	2	
			oL, oC, P	3me	1	1	1	
	pourvu d'un axe principal	d'ordre pair.	A^{2q}, oL^2, oC, oP	4me	$2q$	$2q$		
			A^{2q}, oL^2, C, Π	5me	$2q$	$2q$		
			A^{2q}, qL^2, qL'^2, oC, oP	6me	$4q$			
			A^{2q}, oL^2, oC, qP, qP'	7me	$2q$	1		
			A^{2q}, qL^2, qL'^2, C, Π, qP^2, qP'^2	8me	$2q$	o ou $2q$ (*)		
			A^{2q}, $2qL^2$, oC, $2qP$	9me	$4q$			
		d'ordre impair.	A^{2q+1}, oL^2, oC, oP	10me	$2q+1$	$2q+1$		
			A^{2q+1}, oL^2, C, oP	11me	$4q+2$	$4q+2$		
			A^{2q+1}, oL^2, oC, Π	12me	$2q+1$	$2q+1$		
			A^{2q+1}, $(2q+1)L^2$, oC, oP	13me	$4q+2$			
			A^{2q+1}, oL^2, oC, $(2q+1)P$	14me	$2q+1$	1		
			A^{2q+1}, $(2q+1)L^2$, C, $(2q+1)P^2$	15me	$4q+2$			
			A^{2q+1}, $(2q+1)L^2$, oC, Π, $(2q+1)P$	16me	$2q+1$			
	sphéroédrique	quaterternaire.	$4L^3$, $3L^2$, oC, oP	17me	12			
			$4L^3$, $3L^2$, C, $3P^2$	18me	12			
			$4L^3$, $3L^2$, oC, 6P	19me	4			
			$3L^4$, $4L^3$, $6L^2$, oC, oP	20me	24			
			$3L^4$, $4L^3$, $6L^2$, C, $3P^2$, $6P^2$	21me	6			
		décemternaire.	$6L^5$, $10L^3$, $15L^2$, oC, oP	22me	60			
			$6L^5$, $10L^3$, $15L^2$, C, $15P^2$	23me	12			

(*) Le nombre minimum des sommets de 2me espèce est égal à $2q$, si $q = 1$, et à o, si $q > 1$.

Fig. 6. Classification of polyhedra about their symmetry and minimum number of vertices. (Bravais, 1866)

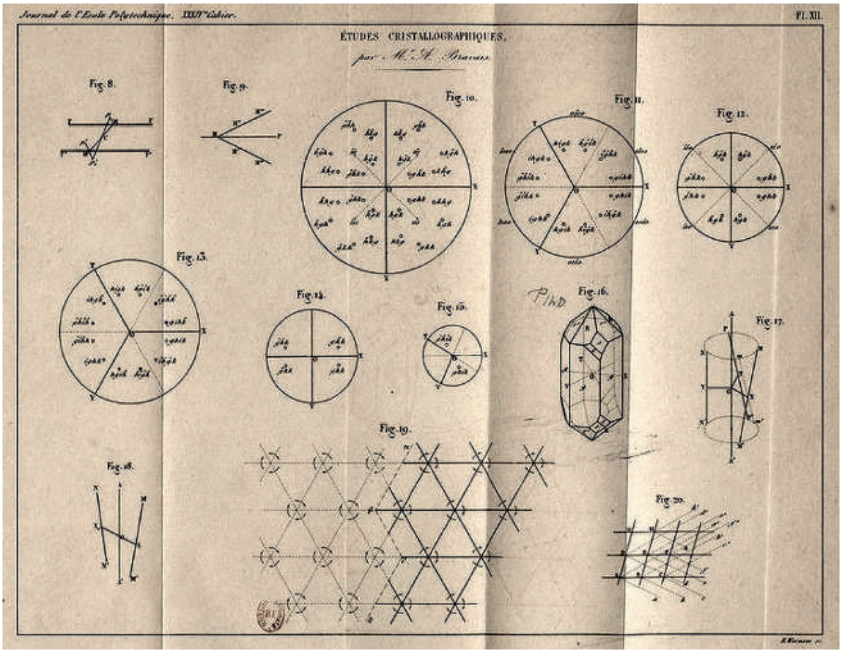


Fig. 7. Compositional scheme of the “crystal latex”. (Bravais, 1851)

These models still effectively provide the space organization according to 3D lattices in which atoms constituting the minerals are located in the nodes. These nodes also represent conjunction elements with further possible lattices according to a geometric structure theoretically repeatable to infinity. These schemes base their geometric structure on the discussed crystallographic group and systems, identifying the possible positions assumed by the atoms and, therefore, the symmetry relationships between the parts. For example, the cubic crystallographic system can correspond to three possible lattice structures depending on whether the atoms are located only in the vertices of the same cube, or even in its center of gravity or in the centroid of the six faces and so also for the tetragonal, trigonal, hexagonal, orthorhombic, monoclinic and triclinic systems different declinations of lattice structures are possible, each characterized by one or more forms of symmetry.

Knowledge of the crystal structure’s atomic-scale organization is fundamental for its influence in determining the characteristics of each precious gemstone. Each of these is characterized by a specific crystallographic system that defines, in theory, and according to models that exclude the presence of impurities and imperfections typical of a natural mineral, the properties of resistance and response to light as about the geometric and dimensional configurations of the molecular structure. These geometric characteristics will partly orient the cutting of the gem according to principles and cutting shapes that will find their foundation in geometry.

GEMSTONES’ DESIGN. SHAPES, SYMMETRIES, PROPORTIONAL RATIO

The geometric rules that regulate the gem cutting find their origins in the configuration of the crystallographic structures. **(Fig. 8)** If the geometrical principles that control the microscopic structure of precious minerals described in the previous paragraph have their foundations based on specific geometrical rules, similarly, the shapes of faceted precious gemstones are managed by rigorous geometric relations. The geometry orients the configurations a crystal can obtain due to manufacturing operations in general and cutting operations in particular. The intrinsic characteristics of a precious mineral, such as color, transparency, and brightness, can be further highlighted by processing operations that will enhance its characteristics by emphasizing the aspects of preciousness (Farrington, 1903). Therefore, the geometrical properties of a faceted gemstone are directly derived from the evolutions of processing techniques over the centuries. The raw crystals, as mined, have, in most cases, opaque external surfaces. In ancient times, precious crystals were used or mounted for ornamentation or artistic processing, and they were extracted, applying the most rudimentary processing techniques. The first techniques were limited to rounding and polishing operations that characterize the so-called “cabochon” cut that gives gemstone a convex and semi-ovoid configuration, still in use mainly in the East with applications to opaque, semi-opaque or transparent and colored stones (Sborgi, 1973). In the fifteenth century, the first shapes of faceted manufacturing were traced, anticipating the cutting systems in use today. The faceted cut is characterized from its first examples by cuts that will affect all the edges of the crystal to enhance the refraction of the incident light, guaranteeing the stone’s brightness. Due to the effects, the facet can ensure, the applications mainly aim at transparent or semi-transparent stones, coloured or not. The classification of the possible shapes of a gem cut stone is systematized in the work “Precious stones: considered in their scientific and artistic relations” by the Anglo-Saxon chemist Arthur Herbert Church (1834-1915). He recognizes cuts with flat and curved surfaces. Cases of hybrid cuts between the two previous classes are rare. Among those with a flat surface, the Church lists and describes ‘Brillant-cut’, ‘Step or trap-cut’, ‘Table-cut’, and ‘Rose cut’; in those with a curved surface, it falls ‘Single cabochon’, ‘Double cabochons’, ‘Hollowed cabochons’, and ‘Tallow tops’. The book by Church, in addition to the textual description of the cuts, combines a graphic apparatus of faceted gemstones starting from the cataloging and redrawing the precious stones of the Townshend Collection at the South Kensington Museum. This collection, which inspired Church’s classification work, is now incorporated into the “Jewellery Gallery” of the Victoria and Albert Museum in London, which houses, also thanks to the donation of the Reverend Chauncey Hare Townshend, one of the complete collections of jewelry and ornaments that also tell the story of the processing of precious stones in Europe (Church, 1905). **(Fig. 9)**

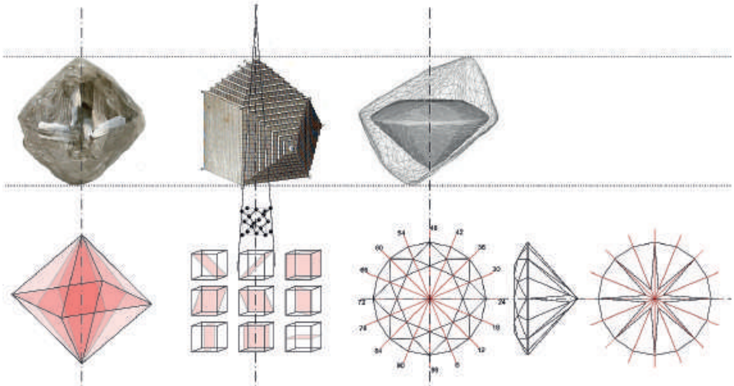


Fig. 8. From the raw structure of the diamond to the gemstone cut. Comparison between the diamond cubic crystallographic structure matrix and the model of the precious stone cut according to the “brilliant cut” (above). The symmetry of the diamond octahedral structure of the raw gem, the minimum cubic unit, is also based on its atomic structure, the cut gemstone (below); Diagram by P. Argenziano, A. Avella, and N. Pisacane, 2024.

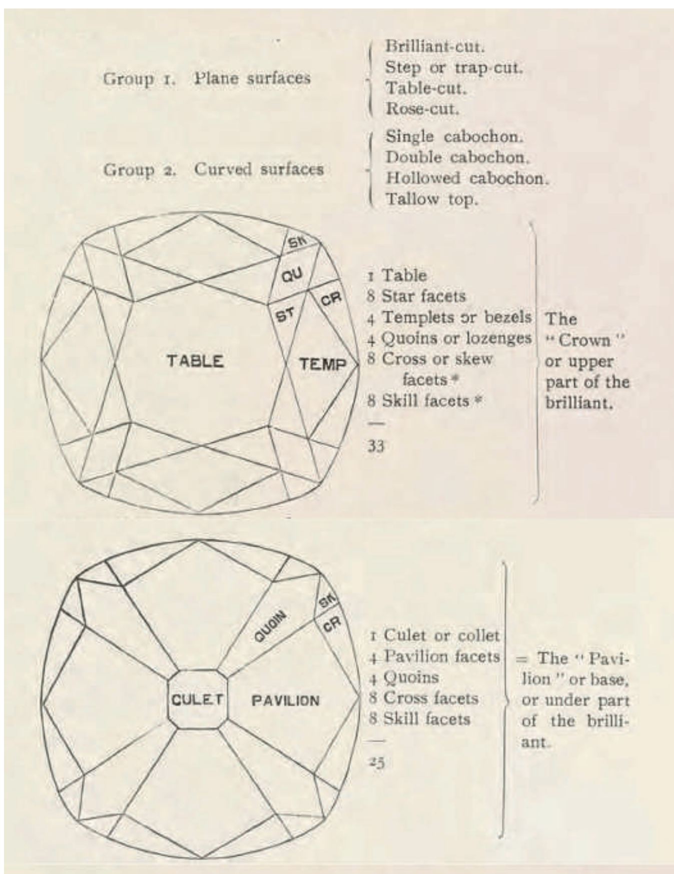


Fig. 9. Classification of different gem cuts according to plane or curved surfaces (above), the parts of upper and downsides of the “brilliant cut” (below). (Church, 1913)

Manufacturing precious stones through facets, starting from the first empirical forms, has been increasingly perfected over the centuries to enhance the intrinsic characteristics of the gemstone itself. Such cutting operations, therefore, highlight the gemstone's qualities in optimizing optical properties and low weight reduction. Luminous performance in terms of brilliance and response to light and limited dispersion of precious material due to cutting, faceting, and polishing operations are the parameters to measure the preciousness of a gem and estimate its value. Brilliance is evaluated as a percentage of the light incident on the gemstone's surface that returns to the outside after subsequent internal reflections. The amount of light that hits a faceted gemstone is partly reflected and refracted. The amount of reflected light directly depends on the refractive index that characterizes a specific material. On the other hand, the amount of refracted light moves inside the gem, changing direction at each incidence with a facet of the same gem and progressively repeating this phenomenon. (Smith, 1912; da Silva et al., 2012)

Cutting a raw stone aims to maximize yield and increase brightness with a reduced loss of carat. Over the centuries, the improvement of cutting models has occurred in parallel with the development of scientific knowledge, directing evolutions in the complexity of geometric configurations of faceted shapes, a function of advances in physics and optics studies, and cutting techniques and technologies. However, geometric principles have always been the invariants that have guided the choice of the position of the cutting planes of the gem facets to increase its ability to refract light and limit the loss of precious material. If, however, in the ancient forms of cutting, the optical, physical, and mechanical properties were little taken into consideration. Subsequently, the progress of studies in the mineralogical sciences has allowed the knowledge of the resistance characteristics of the crystals concerning positions of greater fragility of the stone according to cleavage planes to which the gems are more suitable to be cut, reducing damage during manufacturing. Furthermore, the gem cut design must also consider the critical angle to be respected to obtain the most excellent brilliance from the cut (Mol et al., 2007). The facet constraints must be respected to orient the cutting planes to guarantee that the incident light is reflected an indefinite number of times so that the angles of reflection must have a wider amplitude than that of the limiting angle of the gem itself.

The shape of the faceted gem is the result of optical, technical, and mechanical evaluations, such as determining its configuration. Generally, the parts that make up the faceted gemstone are in well-defined proportional relationships consolidated over time, based on empirical considerations.

Over time, these principles have guided cutting patterns based on geometric and proportionality rules, valid for colored stones and diamonds. Specifically, this type of gemstone-cutting model has been the subject of experimentation and evolution. In the specific case of diamonds, this led to the so-called brilliant cut based on particular rules stated by Tolkowsky in 1919. Marcel Tolkowsky (1899-1991), an engineer by training but belonging to a family of gemstone cutters, combines the scientific approach acquired during his engineering studies with the activities to which his family was dedicated to publishing the book “Diamond Design” in which, despite the merely didactic purpose of the work, he rigorously defines, starting from historical considerations on the processing of precious stones, also supported by principles of optics and mathematics, it describes what is still determined more than a century after its theorizing as the “ideal diamond cut”. The book, as stated in the introduction by the author himself, defines a method of approach to the gem cut design that he experiments with for diamonds, but which can also be extended to colored stones.

It also declares that: *«[…*] The calculations have been made as simple as possible so as not to be beyond the range of readers with a knowledge of elementary geometry, algebra, and trigonometry. However, it was found that the accuracy of the results would be impaired without introducing more advanced mathematics, so these have been used, and graphical methods have been explained as an alternative. The results of the calculations for the form of brilliant now in use were verified by actual measurements from well-cut brilliants. The measures of these brilliants are given at the end of the volume in a tabulated and graphical form. It will be seen how strikingly near the actual measures are to the calculated ones […]» (Tolkowsky, 1919, p. 6), underlining both the role of geometry in the configuring processes of the shape and the possibility of an exclusively graphic and alternative approach to the mathematical one for solving the problems of defining the best cut.

The possible configurations of refraction of the light ray and the configuration and position of the facets are clearly illustrated in the tables attached to the volume or the illustrations accompanying the text. A graphical and comparative analysis of the crown part of three diamond cuts used at the beginning of the 20th century introduces the “ideal diamond cut” design. This study is the premise of the “ideal diamond cut” design illustrated in double orthogonal views from the top and bottom sides of the gemstone and a vertical section in which different proportional ratios between parts are indicated. The dimensions of the various parts all refer to the maximum diameter of the faceted stone (belt), which, assuming a value of 100, defines the diameter of the table of 53, the height of the pavilion at 43.1, and that of the crown at 16.2. The drawing of the brilliant cut in orthogonal projection from above and below clarifies the articulation of the shape. Finally, through geometric evaluations on the reflection of light, 40°45’ is defined as the best angle between the facets of the pavilion and the horizontal plane of the belt, while between the aspects of the crown and the plane described above, 34°30’. The vertical section of the gemstone also allows a graphical solution through the representation of an incident ray to get the different conditions of light behavior. **(Figg. 10, 11)**

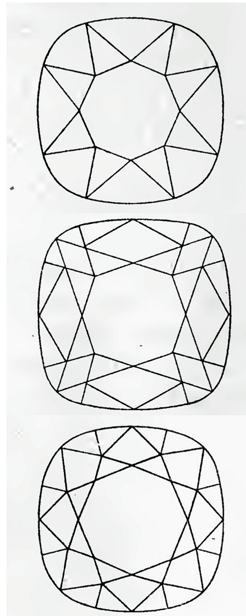


Fig. 10. Comparison in top orthogonal projections between diamond cuts: Mazarins diamond cut with 16 facets (above), Peruzzi diamond cut with 32 facets (middle), old-brilliant cut with 32 facets. (Tolkowsky, 1919)

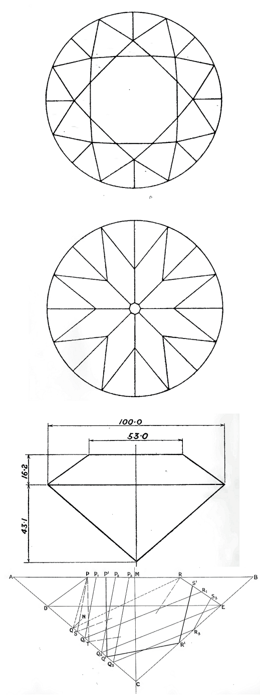


Fig. 11. Double orthogonal projections of “ideal diamond cut” (above), a vertical section with the proportional ratio between parts (middle), and a vertical section with incident lights behavior. (Tolkowsky, 1919)

The execution of the cut that takes its name from Tolkowsky himself, as he stated in the historical notes in the introduction to the book, also bases its theorization on the evolution and refinements through the centuries of the devices for faceting and polishing. (Bycroft & Dupré, 2019). The most complex cutting morphologies will take place from the beginning of the nineteenth century with the introduction of 'jambpeg' technology, which combines the cutting devices with a system for the perfect definition of the position of the faceting planes, overcoming the limits of the tools used so far which provided for the smoothing of the rough stone with purely empirical methods. (Prim, 2021) At the beginning of the 1900s, at a time coeval with Tolkowsky's work, the first cutting machines with a vertical shaft and protractor appeared that, through a graduated scale, allowed perfect control of the shape. Although further improved, this instrumentation is still in use and only for diamonds, replaced by laser cutting techniques through control of the manufacturing processes based on digital models. **(Fig. 12)**

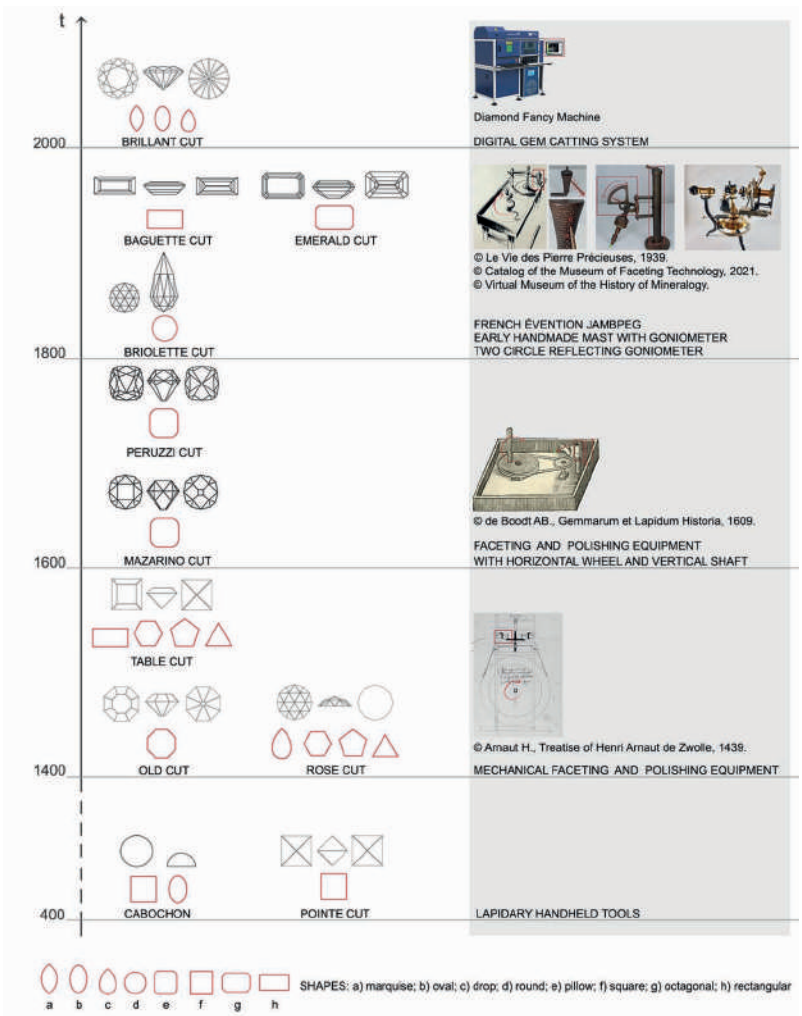


Fig. 12. Chronological diagram of types, shapes, and cutting tools; Diagram by Francesca Fabozzi, 2022.

CRYSTALLOGRAPHIC FORMS' MODELS, FROM TANGIBLE TO DIGITAL

by Alessandra Avella

FROM THE MICROSCOPIC STRUCTURE OF CRYSTALS TO CRYSTALLOGRAPHIC FORMS' MACROSCOPIC MODELS

The geometric matrices of crystals and their modeling depend on the intrinsic chemical characteristics of the constituent material and on how atoms naturally aggregate in microscopic space. In contrast, the environment in which a crystal develops can only influence its size and its regularity (crystal habit). These characteristics of crystals, already intuited in the second half of the seventeenth century by Niels Stensen (1638-1686) and confirmed from the beginning of the twentieth century with the advent of X-ray diffraction (Mascarenhas, 2020), make them in nature more comparable than others to abstract polyhedra, theorized since ancient times. Macroscopic and tangible models of crystallographic forms have been used since the first steps of morphological crystallography coined by Romé de L'Isle (1736-1790) (Romé de L'Isle, 1783) and René Just Haüy (1743-1822) (Haüy, 1784, 1801, 1817, 1822) to overcome the difficulties of having to deal with geometrical aspects that appear abstract given microscopic dimensions of crystals, despite being objectively natural.

De L'Isle is credited with being the first to intuit that the tangible model of a crystal could macroscopically reproduce the polyhedral form of its primitive unitary cell, later defined by Haüy as *molécule intégrante*. The first series of crystallographic forms' tangible models date back to the end of the Eighteenth century when the French mineralogist himself had a collection of 395 small (ca 3 cm) terra cotta models[1] made as a tangible support for his treatise *Cristallographie, ou description des formes pro-pres a tous les corps du regne mineral*, published in Paris in 1783 **(Fig. 1)**. This treatise is in four volumes with thirty-two analytical cards, and twelve plates; In these plates, for the first time, crystals are graphically systematized as morphological declinations starting from regular geometric. (Touret, 2004) The author was the first to introduce the experimental method in the morphological study of crystals, natural and artificial, providing the first theoretical and graphic classification, and generalized to all species of crystals the law of the constancy of the dihedral angle (Stensen, 1669) had enunciated concerning some specific cases.

1 The terra cotta models were bought in Paris in 1785 by Martinus van Marum, the first director of the Teylers Museum in Haarlem, the Netherlands. The complete collection still resides in the Museum.

Fig. 1. Rome de l'Isle, terra cotta models: gypsum (top) and staurolite (botton). (Collection, Teylers Museum, Haarlem, the Netherlands, 2024).

About twenty years later, the French Abbot René Just Haüy, founder of scientific crystallography, took up and improved the Romé de L'Isle's analytical approach by taking as a reference the collection of minerals of the *Ecole de Mines*, of which he was conservative: he related the mineralogical species with the polyhedral forms of crystals, drawn as diaphanous solids ("solide diaphane") "using the method of projection, assuming the point of view far to infinity" (axonometry) (see Chapter 3). All the crystal polyhedra, represented in 8 plates and 31 synoptic tables in the volume of the atlas of his *Traité de minéralogie* (1801), were concretized in pear wood models (sizes between ca 2.5 and 10 cm) by Pleuvin and Journy (Haüy, 1801). Haüy was the first to recognize the superiority of wood over clay, which had been used until then for the construction of tangible models; In particular, he adopted the pear wood, making it possible to obtain smooth faces, sharp edges and dihedral angles of the models more precisely. Today, 565 of these models are still preserved in the Teylers Museum in Haarlem, the Netherlands, making this collection of wooden specimens the most complete of Haüy's surviving collections **(Fig. 2)**.

Fig. 2. R. J. Haüy, pear wood models: titanite crystal and quartz crystal (top from left to right); one of the five drawers housing the collection (bottom) (Collection, Teylers Museum, Haarlem, the Netherlands, 2024).

As technical advances and the development of scientific studies in crystallographic matters progressed, the quality of the production of models to support treatises increased considerably, and several mineralogists and crystallographers of the time designed their series. Since the beginning of the nineteenth century, having ascertained the intrinsic communicative efficacy of the crystallographic forms' tangible models, they continued to be made not only to support research and to improve understanding of treatises, but also to teach new generations the principles of crystallography and to illustrate to students the abstractions of Solids Geometry, according to a sensory approach to learning, which cannot be separated from its formal perception; beginning with de L'Isle, the crystallographic form's tangible models materialize abstract polyhedral - in the Kantian, they give them a 'form' to become aware of them - and circumvent the problems of their graphic representation, then fully solved at the beginning of the Nineteenth century with the contribution of Projective and Descriptive Geometries applied to crystallography.

Given these years' great scientific and cultural success, the reference to the Swiss pedagogue and philosopher Pestalozzi (1746-1827) is particularly significant in framing the cultural context in which the increase in the production of crystallographic forms' models for educational use is considerable.

In his *Anschauungslehre der Zahlenverhältnisse* (1803) (Pestalozzi, 1803), Pestalozzi confirms that the 'representation' of abstract concepts, especially geometric-mathematical ones, plays a central role in facilitating and promoting cognitive processes, translating what is 'abstract' into what can be experienced through the senses. Affirming that everything, even the most abstract thing, takes shape in the physical world and the plastic world, the Swiss philosopher leads the reasoning «around images and communicative artifacts that, in their articulation, decline in various ways the idea of 'transcription' of complex theories from one language (that of abstract logic) to another (that of sensible experience), to make the noetic process faster, simple and precise». (Cirafici, 2022, p. 198).

According to Pestalozzi, the transcription of in-formation, meaning 'giving form' to a specific datum or a data set, means making complex and abstract theoretical principles visible and therefore comprehensible through concrete tangible or graphic representations, whose undoubted pedagogical usefulness is recognized. It is a matter of relating abstract mathematical thought with aesthetic practices according to an approach that from Pestalozzi manifests itself in the artistic experiments of the Avant-gardes between the end of the Nineteenth and the beginning of the Twentieth century: from the artistic work by Dutch painter Piet Mondrian (1872-1944), in which mathematics enters the 'visible', to the one in the architectural field by Ozenfant (1886-1966) and Le Corbusier (1887-1965), in which mathematical thought is translated into a "tangible concept".

Therefore, Pestalozzi and the contemporary German philosopher Froebel (1782-1852) are recognized as the precursors of both pedagogical theories of the Bauhaus, that inspired the thought of the artistic avantgardes and of the Modern Movement - of which the Bauhaus professors were leading exponents - and of the 'Montessori Method' (Montessori, 1909), an interesting pedagogical experience of the late Nineteenth century that inaugurated a phenomenological approach to learning. This approach proceeds with 'development materials' designed as a teaching support, especially in mathematics and geometry (Anceschi, 2009, p. 16).

Within the context of the reflections that have been proposed here on the cultural experiences that have stimulated the production of crystallographic forms' tangible models as 'materialized abstractions' for educational purposes, the 'development materials' by Montessori (1870-1952), as well as the 'coloured' book *The Elements of Euclid* (1847) (Byrne, 1847) by Oliver Byrne (1810-1880), represent some emblematic examples.

The Montessori materials for development of developing mathematical and geometric thought are 'communicative artefacts', which have a powerful force of logical seduction that supports learners in conceiving and representing complex logical-spatial systems. In addition, they have an undoubted aesthetic value **(Fig. 3)**.

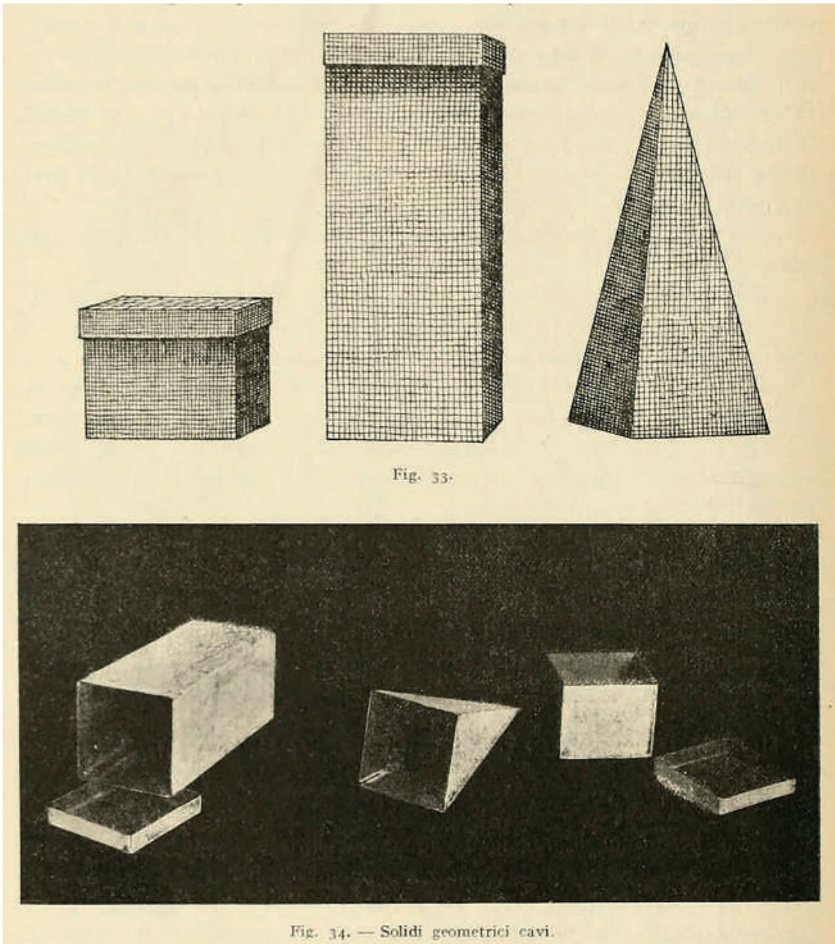
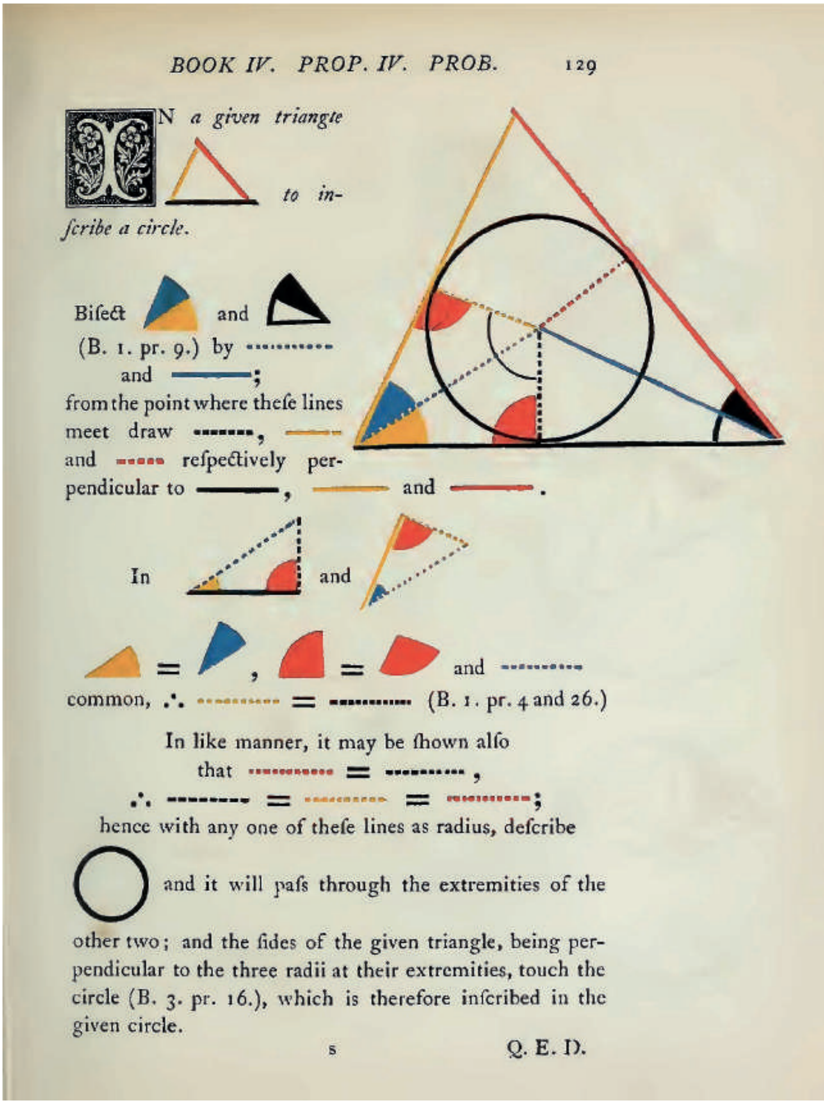


Fig. 3. Educational materials by Montessori: hollow geometric solids, drawing and models. (Montessori, 1916, p. 460)

In his *The Elements of Euclid*, Byrne moves from theoretic geometry to its 'sensible transcription', transforming Euclid's statements into shapes, colours and symbols to make complex reasoning 'visible' and therefore understandable.

The two scholars are interpreters of an entire culture that recognizes in tangible and figurative representation an accelerator of the learning processes of abstract geometric-mathematical thought for their intrinsic ability to build connections of meaning between what Einstein defined as 'conceptual schemes devoid of content'[2] and the reality **(Fig. 4)**.

2 Albert Einstein's statement in his famous lecture "Geometry and Experience" at the Austrian Academy of Visual Studies on January 27, 1921.


BOOK IV. PROP. IV. PROB. 129

IN a given triangle to inscribe a circle.

Bisect and (B. 1. pr. 9.) by and ; from the point where these lines meet draw , and respectively perpendicular to , and .

In and

= , = and common, ∴ = (B. 1. pr. 4 and 26.)

In like manner, it may be shown also that = , ∴ = = ; hence with any one of these lines as radius, describe and it will pass through the extremities of the other two; and the sides of the given triangle, being perpendicular to the three radii at their extremities, touch the circle (B. 3. pr. 16.), which is therefore inscribed in the given circle.

s Q. E. D.

Fig. 4. Book IV Prop. IV, “In a given triangle to inscribe a circle”. (Byrne, 1847, p. 129)

By extending the pedagogical experience of the German school of Pestalozzi and Froebel and its cultural context to the crystallographic field, it is possible to trace the prodromes of the didactic character of the articulated and numerous collections of crystallographic forms’ tangible models produced from the end of the eighteenth century to facilitate the understanding of complex solids and their geometric characteristics. If pear wood, among the various essences experimented, is the most used material to improve the geometric precision of crystallographic forms’ models for educational use, models in plaster, cast iron, lead, brass, porcelain, and glass are also produced. (Schuh, 1984)

In particular, glass models were made to represent the intrinsic geometric characteristics of the crystals; while the development of polyhedra on cardboard to be cut out was adopted to reduce production and shipping costs and to expand the purchasing people. The first cardboard and glass models are attributed to the mineralogist Adam August Krantz (1808-1872), founder of the homonymous German mineral dealership, scientific instruments, and teaching aids, expanded by his nephew Friedrich L. R. Krantz (1859-1926) thanks to the collaboration with Karel Vrba (1846-1923), Professor of Mineralogy at the Bohemian University of Prague ('Karel Vrba', 1923). The optimization of the glass models with the insertion of colored threads to highlight the crystallographic axes and of the hemihedral polyhedral in cardboard, translating Haüy's "diaphanous solids" into a tangible form, is to be attributed to Czech mineralogy (Krantz, 1910)

The publishing house Paravia (Turin) was the Italian competitor of the German dealership; Over the course of the Twentieth century, it produced various laboratory instruments and educational aids made in Italy[3], for various scientific subjects in schools and universities, adapting its supply of educational materials to the different cultural and political changes in Italy and the school system.

Among the materials and educational aids for the teaching of Natural Sciences published in the Paravia Catalogs between 1917 and 1987, several collections of crystallographic forms' models are offered for sale in the section relating to mineralogy, geology and crystallography. The wall Tables of the simple and derived forms of the crystallographic systems and the development of polyhedra on cardboard are present only in the 1917 Catalog, in addition to the tangible models.

Crystallographic forms' models in wood and glass made by Paravia are very rare **(Fig. 5)**; the collections preserved in the Royal Mineralogical Museum of the University of Naples Federico II and the Museum of Earth Sciences of the Department of Earth and Geoenvironmental Sciences of the University of Bari[4] are among the most copious; In addition, the Michelangelo Museum in Caserta[5], established in 2004 by Istituto Tecnico Statale per Geometri "Buonarroti", preserves the collection of 15 glass specimens exemplifying the fundamental crystallographic forms within the largest collection of natural objects, scientific instruments, technological apparatuses and didactic-scientific models. (Di Lorenzo, 2018, 2020) **(Fig. 6)**

3 The crystallographic forms' models, put on the market since the beginning of the Twentieth century by Paravia, reach a level of technical and scientific precision such as to meet the requirements set by the National Government with Royal Decree no. 527 of 1927 regarding national production. For this reason, in 1936, Paravia's products obtained the "recognition of completely Italian material" (prot. N. 5598/36) by the Ministry of Corporations, competing for accuracy and price with the production of foreign companies, which for decades had monopolized the market of teaching aids in Italy and in European countries, probably also inspiring the production of Paravia during the early years.

4 The 20 specimens of crystallographic forms' models in the latter collection are cataloged and archived in the General Catalogue of Cultural Heritage (ICCD, n.d.)

5 Since 2009, the Michelangelo Museum has been a member of the "Terra di Lavoro" Museum System. The Museum's collections are constantly increasing, also thanks to private donations; The exhibition is organized in the sections: Mineralogy, Topography, History of Measurement, Calculus, Writing, Mechanics, Audio-Video Recording, History of Drawing, Pure Sciences and Garden of Mathematical Machines. The collection of the 15 glass specimens, on display in the Mineralogy section, was acquired by the "Buonarroti" Institute in 1963.

Fig. 5. Set of 52 crystal forms' wooden models, Michelangelo Museum Collection, Caserta, Italy. Photos by P. Argenziano, A. Avella, and N. Pisacane, 2024.

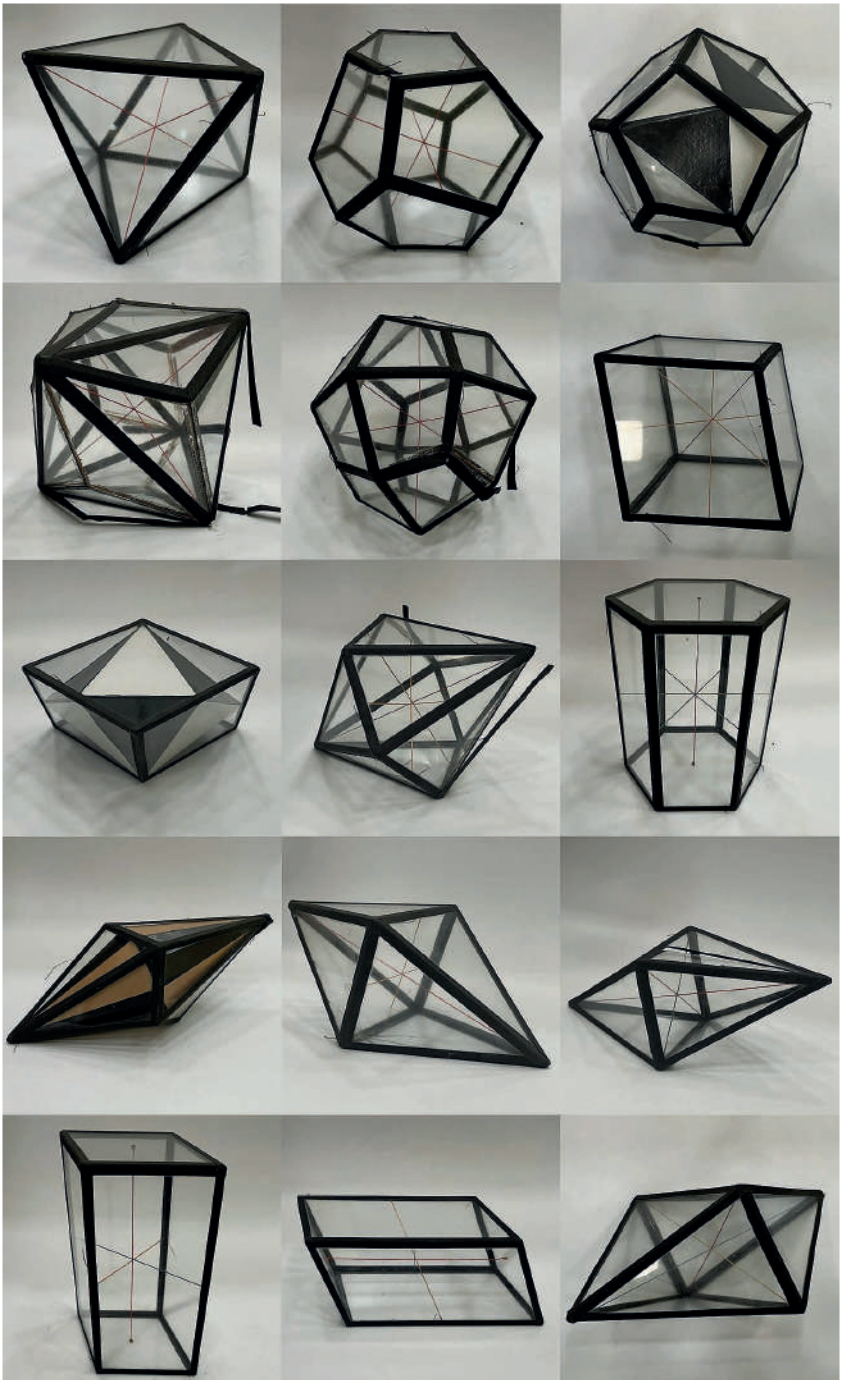


Fig. 6. Set of 15 crystallographic forms' glass models with crystallographic axes in colored silk thread and hemihedral forms in cardboard by "G.B. Paravia & C." company. Michelangelo Museum Collection, Caserta. Photos by P. Argenziano, A. Avella, and N. Pisacane, 2024.

In addition, to the crystallographic ones, the 35 architectural models are noteworthy for their perfect state of preservation[6] made by the V. Toffoli & Figli company[7]. **(Fig. 7)**

Fig. 7. Tangible models of architectural orders (top) and sectioned tangible model of a building (bottom), from a set of 35 by V. Toffoli & Figli company. Michelangelo Museum Collection, Caserta, Italy. Photos by the Authors, 2024

6 On which in-depth studies are underway by the authors of this volume.

7 The collection of 35 architectural models is of great interest for the precision and detail of the realization; it includes the five architectural orders, the moldings for the decorations, the portals, the windows, the rose windows of the Romanesque, Gothic, Renaissance, Baroque periods, etc., all datable to the period 1920-1930. From its foundation in 1947 to its closure in 1972, the V. Toffoli & Figli company dominated the market of educational models with a catalogue of about three thousand items, which is always updated.

Taking up Rome de L'Isle's intuition, studying of the 15 glass specimens of the Michelangelo Museum[8] has made it possible to systematize the fundamental crystallographic forms and underlying morphological, geometric and parametric relationships already discussed in the previous chapter (see Chapter 1).

These forms have been identified according to their respective geometric characteristics about several faces of the polyhedron, the forms of the individual polygons constituting the faces and their reciprocal position in space, dihedral angles between the same faces, planes and axes of symmetry. This is because the attribution of each specimen of the 15 models to their respective crystallographic form is lacking. The following crystallographic forms have been identified: an octahedron, a pentagon-dodecahedron, a pentagon-dodecahedron that includes a cardboard tetracysesahedron, a triacisoctahedron, and an icositetrahedron. These forms belong to the cubic system because they have three mutually orthogonal crystallographic axes: a rhombohedron, a rhombohedron that includes a cardboard hexagonal pyramid, a hexagonal bipyramid, a hexagonal prism and a scalenohedron. These forms belong to the hexagonal system because they have a hexagonal form in the same polyhedron or its hemihedral form; a squaretottahedron that belongs to the tetragonal or dimetric system because the triangular faces of the two pyramids have a common square base; a rhombottahedron and a rhombic prism. These forms belong to the homonymous system for the rhombic form of the base polygon; a monoclinic prism or oblique prism with a rhomboidal base. This form belongs to the monoclinic system because the direction of the edges is not orthogonal; a triclinic octahedron or oblique double pyramid with a rhomboidal base. These forms belong to the triclinic system because they do not respect any geometric principle found in the above forms (Haüy, 1784).

The results of the phase of identification of the crystallographic forms of the 15 glass specimens have been systematized in an *ad hoc* structured Table that orders and classifies them **(Figg. 8, 9, 10)** attributing each of these forms to the group, the system and the crystallographic class to which they belong.

8 On 2024, May 20th a general agreement has been signed between the Department of Architecture and Industrial Design of the University of Campania Luigi Vanvitelli, and Museo Michelangelo/Istituto Tecnico Statale in Caserta (Italy); the scientific coordinators are P. Argenziano, A. Avella, N. Pisacane.

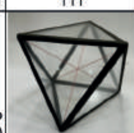
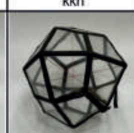
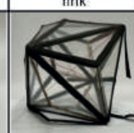
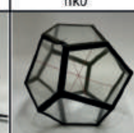
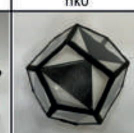
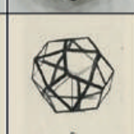
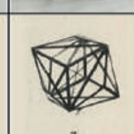
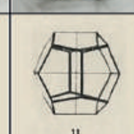
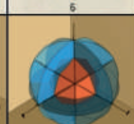
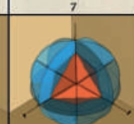
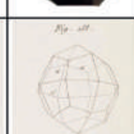
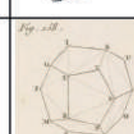
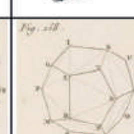
GROUP (3)	*Monometric/ Spherical*	*Monometric/ Spherical*	*Monometric/ Spherical*	*Monometric/ Spherical*	*Monometric/ Spherical*
SYSTEM (7)	*Cubic*	*Cubic*	*Cubic*	*Cubic*	*Cubic*
SYSTEM (6)	Cubico o tessulare	Cubico o tessulare	Cubico o tessulare	Cubico o tessulare	Cubico o tessulare
CLASS (32)	Esacisottaedrica	Esacisottaedrica	Esacisottaedrica	Diacisdodecaedrica	Diacisdodecaedrica
CRYSTAL FORM (from 3DCrystalroom software)	Ottaedro	Trapezoedro	Triacisottaedro	Pentagonododeca edro	Pentagonododeca edro
CRYSTAL FORM (glass model)	Ottaedro	Icositetraedro	Triacisottaedro	Pentagono-dodecaedro	Pentagono-dodecaedro
HEMIHEDRAL FORM (paper)					Tetracisesaedro
CRYSTAL FORMS (from Paravia catalogue, 1968)	Ottaedro con assi interni	Icositettraedro con assi interni	Triacisottaedro con assi interni	Pentagono dodecaedro con assi interni	Pentagono dodecaedro con tetracisesaedro incluso
MINERALOGIC SPECIES	Allume, ferro magnetico	Granato	Galena	Pirite di ferro, cobaltina	Pirite di ferro, cobaltina
CRYSTAL AXIS	● ● ●	● ● ●	● ● ●	● ● ●	
Numbers of faces (S)	8	24	24	12	12
Numbers of edges (E)	12	48	36	30	30
Numbers of vertexes (V)	6	26	14	20	20
Number of sides limiting faces (n)	3	4	3	5	5
Symmetry center	si	si	si	si	si
Simmetry axis in class 2	6	6	6	3	3
Simmetry axis in class 3	4	4	4	4	4
Simmetry axis in class 4	3	3	3	no	no
Simmetry axis in class 6	no	no	no	no	no
Simmetry planes	9	9	9	3	3
ID Miller	111	kkh	hhk	hk0	hk0
CRYSTAL FORMS (Museo Michelangelo collection)					
CRYSTAL FORMS (from Paravia catalogue, 1968)					
CRYSTAL FORMS MODELS WITH CRYSTAL AXIS AND SYMMETRY PLANES (Software 3DCrystalroom)					
MINERALOGIC SPECIES					
CRYSTAL FORMS OF AXONOMETRIC PROJECTIONS (Hauy, 1801)					
POSITION (Hauy, 1801)	pl. XXXVIII fig. 147	pl. XXXIX fig. 158		pl. LXXVIII fig. 158	pl. LXXVIII fig. 158

Fig. 8. Monometric group crystallographic forms' glass models, on display at Michelangelo Museum in Caserta. Classification of the 5 specimens. Table by P. Argenziano, A. Avella, N. Pisacane, 2024.

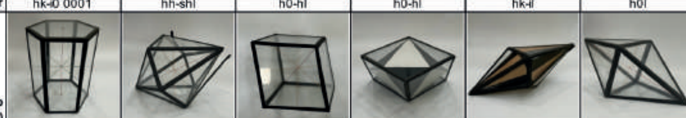
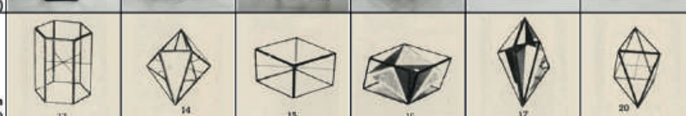
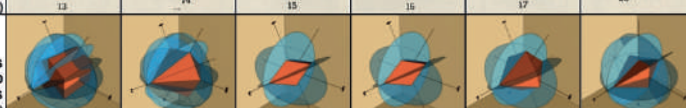
GROUP (3)	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*	*Dimetric/ 2 axis ellipsoid/ ellipsoid of revolution*
SYSTEM (7)	*Hexagonal*	*Hexagonal*	*Trigonal*	*Trigonal*	*Trigonal*	*Tetragonal*
SYSTEM (6)	Esagonale	Esagonale	Esagonale	Esagonale	Esagonale	Tetragonale
CLASS (32)	Bipiramidale diesagonale	Bipiramidale diesagonale	Scalenoedrica ditrigonale	Scalenoedrica ditrigonale	Scalenoedrica ditrigonale	Bipiramidale ditetragonale
CRYSTAL FORM (from 3DCrystalroom software)	Prismaesagonale Pinacoide	Bipiramideresagonale	Romboedro	Romboedro	Scalenoedro ditrigonale	Bipiramidetetragonale
CRYSTAL FORM (glass model)	Prisma esagonale	Bipiramide esagonale	Romboedro	Romboedro	Scalenoedro	Quadratottaedro
HEMIHEDRAL FORM (paper)				bipiramide esagonale	Bipiramide dodecagonale	
CRYSTAL FORMS (from Paravia catalogue, 1968)	Prisma esagonale	Bipiramide esagonale	Romboedro con assi interni	Romboedro con piramide esagonale inclusa	Scalenoedro con bipiramide inclusa	Bipiramide tetragonale con assi interni
MINERALOGIC SPECIES	Spato calcare, apatite	Quarzo, piromorfite	Spato calcare	Spato calcare	Spato calcare	Anastasio
CRYSTAL AXIS	● ● ● ●	● ● ● ●	● ● ● ●			● ● ●
Numbers of faces (S)	8	12	6	6	12	8
Numbers of edges (E)	18	18	12	12	18	12
Numbers of vertexes (V)	12	8	8	8	8	6
Number of sides limiting faces (n)	6/4*	3	4	4	3	3
Symmetry center	si	si	si	si	si	si
Simmetry axis in class 2	6	6	3	3	3	4
Simmetry axis in class 3	no	no	1	1	1	no
Simmetry axis in class 4	no	no	no	no	no	1
Simmetry axis in class 6	1	1	no	no	no	no
Simmetry planes	7	7	3	3	3	5
ID Miller	hk-i0 0001	hh-shl	h0-hl	h0-hl	hk-il	h0l
CRYSTAL FORMS (Museo Michelangelo collection)						
CRYSTAL FORMS (from Paravia catalogue, 1968)						
CRYSTAL FORMS MODELS WITH CRYSTAL AXIS AND SYMMETRY PLANES (Software 3DCrystalroom)						
MINERALOGIC SPECIES						
CRYSTAL FORMS OF AXONOMETRIC PROJECTIONS (Hauy, 1801)						
POSITION (Hauy, 1801)	pl. XXXVIII fig. 149	pl.XL fig. 1	pl. XXIII fig. 7	pl. XXIII fig. 7	pl. XXVIII fig. 51	pl. LVII fig. 167

Fig. 9. Dimetric group crystallographic forms' glass models, on display at Michelangelo Museum in Caserta. Classification of the 6 specimens. Table by P. Argenziano, A. Avella, N. Pisacane, 2024.

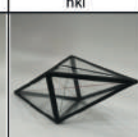
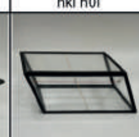
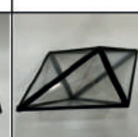
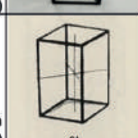
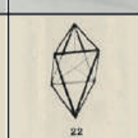
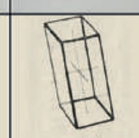
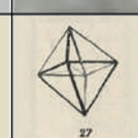
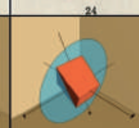
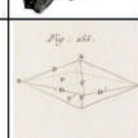
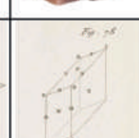
GROUP (3)	*Trimetric/ 3 axis ellipsoid/ scalene ellipsoid*	*Trimetric/ 3 axis ellipsoid/ scalene ellipsoid*	*Trimetric/ 3 axis ellipsoid/ scalene ellipsoid*	*Trimetric/ 3 axis ellipsoid/ scalene ellipsoid*
SYSTEM (7)	*Orthorhombic*	*Orthorhombic*	*Monoclinic*	*Triclinic*
SYSTEM (6)	Rombico	Rombico	Monoclino	Triclinico
CLASS (32)	Bipiramidale rombica	Bipiramidale rombica	Prismatica	Pinacoidale
CRYSTAL FORM (from 3DCrystalroom software)	Prisma Pinacoide	Bipiramiderombica	Prisma Pinacoide	
CRYSTAL FORM (glass model)	Prisma rombico	Rombottaedro	Prisma monoclino	Ottaedro triclino
HEMIHEDRAL FORM (paper)				
CRYSTAL FORMS (from Paravia catalogue, 1968)	Prisma rombico	Bipiramidale rombica	Pinacoidi del monoclino con assi interni	Combinazione di quattro pinacoidi di quarta specie con assi interni
MINERALOGIC SPECIES	Salnitro	Zolfo	Soda	Vetriolo azzurro
CRYSTAL AXIS	● ● ●	● ● ●	● ● ●	● ● ●
Numbers of faces (S)	6	8	6	8
Numbers of edges (E)	12	12	12	12
Numbers of vertexes (V)	8	6	8	6
Number of sides limiting faces (n)	4	3	4	3
Symmetry center	si	si	si	si
Simmetry axis in class 2	3	3	1	no
Simmetry axis in class 3	no	no	no	no
Simmetry axis in class 4	no	no	no	no
Simmetry axis in class 6	no	no	no	no
Simmetry planes	3	3	1	0
ID Miller	hk0 001	hkl	hkl h0l	
CRYSTAL FORMS (Museo Michelangelo collection)				
CRYSTAL FORMS (from Paravia catalogue, 1968)				
CRYSTAL FORMS MODELS WITH CRYSTAL AXIS AND SYMMETRY PLANES (Software 3DCrystalroom)				
MINERALOGIC SPECIES				
CRYSTAL FORMS OF AXONOMETRIC PROJECTIONS (Hauy, 1801)				
POSITION (Hauy, 1801)		pl. XXXIX fig. 155	pl. XLVIII fig. 78	

Fig. 10. Trimetric group crystallographic forms' glass models, on display at Michelangelo Museum in Caserta. Classification of the 4 specimens. Table by P. Argenziano, A. Avella, N. Pisacane, 2024.

In the glass specimens, forms belonging to the three crystallographic groups[9] and to all crystallographic systems are identifiable. The classification of systems refers to the polyhedral form of the primitive unit cell, concerning which the atoms of each crystal are aggregated. According to this classification, it should be emphasized that Paravia's crystallographic forms' models are divided into six systems and not into seven[10]. This division into six systems, which merges the trigonal and hexagonal systems, was probably proposed by Paravia for didactic simplification. In this regard, it is interesting to point out that among the educational aids produced by Paravia, starting from the 1932 catalog, there are also available "the six fundamental forms of crystallization systems – Painted iron wire models: in red the corners, in white the axes". The analysis of the image of these models, included in the catalog, shows that the trigonal system is excluded from the crystallographic systems **(Fig. 11)**.

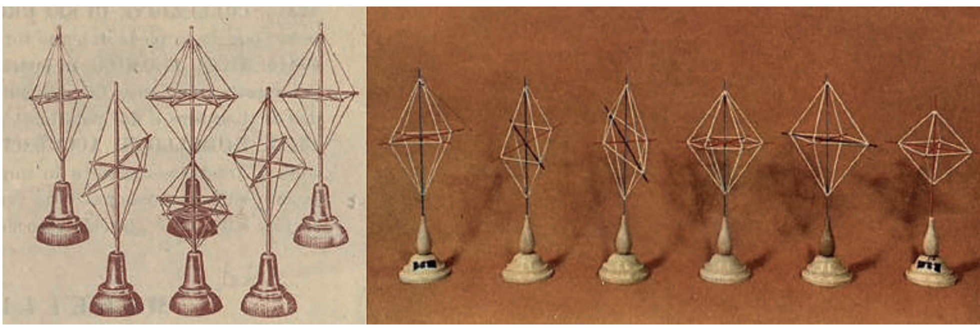


Fig. 11. The six basic forms of crystallization systems. Sepia drawing (top) (Paravia, 1951, p. 65); Colour photograph (bottom) (Paravia, 1980, p. 146).

In addition, the presence of silk thread axes, their number and colour according to the crystallographic group or the respective hemihedral cardboard form are indicated in the Table for each crystallographic form's model.

Silk threads of one, two or three colours that materialize the direction of the crystallographic axes are found in twelve glass specimens among the fifteen ones on display at the Michelangelo Museum, as described below:

- the forms belonging to the monometric group have red monochromatic threads, mutually orthogonal, whose intersections identify the center of the crystal;

- the forms of the dimetric group have double-coloured threads, in red/yellow or red/green colors. Specifically, the models belonging to the tetragonal system have one red wire and two green wires, while the models belonging to the hexagonal system have one red wire and three yellow ones, incidental to each other, coplanar and of equal length;

9 Crystallographic forms are divided into monometric, dimetric and trimetric groups, so called in relation to the angles that the crystallographic axes mutually form with each other

10 The literature currently identifies the seven crystal systems – cubic, hexagonal, tetragonal, rhombic, monoclinic, triclinic and trigonal – according to the symmetry classes

- the forms belonging to the trimetric group have threads of three colours (blue/yellow/red or green/yellow/red), incident in the center of the crystal, always forming angles other than the right one;

Only three specimens, among the fifteen ones, have the complementary hemihedral form in cardboard inscribed in the glass polyhedron. Among the monometric system model, the pentagon-dodecahedron (12 faces) includes the tetrakis-hexahedron (24 faces) with triangular faces alternately in black or white: the black ones are coincident with the glass polyhedron unlike the white ones. Among the dimetric system model, the rhombohedron (6 faces) includes a double pyramid with a hexagonal base (12 faces). It is classifiable in the hexagonal hemihedry, while the scalenohedron (12 faces) includes a double pyramid with a dodecagonal base (24 faces) in the rhombohedral hemihedry (Sansoni, 1892). In the case of the rhombohedron, the white triangular faces of the hexagonal bipyramid in paper are all coincident with the glass faces, unlike the black ones.

These specimens are examples of some hemihedral forms, which allow some basic forms to be derived from others, halving the number of faces (Sansoni, 1892). About the pentagon-dodecahedron and the rhombohedron, the comparison with the homologous models with silk thread axes, also available in the Museum’s collection, certainly provided a useful teaching aid and allowed further geometric considerations on these crystallographic forms.

The Table also compares the glass models on display in Caserta with those shown in the Paravia catalogs (with specific reference to the 1932 edition) using photographic images of the models and illustrations included in the catalog, to geometrically verify each form and hypothesize the dating of the 15 specimens on display at the Michelangelo Museum.

Preparatory activity for the comparative analysis aimed at a hypothesis of chronological attribution of the models was the systematization of the collections of crystallographic forms’ models produced by Paravia according to the chronological order of publication in the Catalogs of Natural Sciences edited between 1917 and 1987[11] **(Figg. 12, 13).**

11 The iconographic apparatus, which illustrates the Collections or the individual models, follows the typographic layout of the series to which the Catalog belongs. In the Catalogs, published from 1917 to 1942, the images, mainly drawings and photographs in black and white, are inserted like the text in the layout grid; in the 1951 Catalog, which belongs to the “Cataloghi della Rinascita” Series, the typographic layout does not change, but the drawings and photographs are sepia-toned. In the following catalogs, the images are composed on double-sided colour or black and white plates, on pages outside the text. The last two editions of 1980 and 1987 had a new typographic layout designed to facilitate the consultation of the products on sale, with the illustrations almost always in colour laid out in direct correspondence with the descriptive texts. The detail of the images of the models presented is treated with particular attention also to avoid any sampling, i.e. the shipment on request of the items as a sample for choice.

COLLECTION/NAME	PIECES MATERIAL	NUMBER OF PIECES	PIECE SIZE	DESCRIPTION	CONTAINER BOX	IMAGE	PAGE	PRICE	CATEGORY	SECTION/CHAPTER	TITLE	DATE
Collection of 52 pieces according to ROTHE	wood	52		NO	NO	NO	66	L.60	Mineralogy, Geology and Paleontology	Models of crystallographic shapes	Materiale e sussidi didattici per l'insegnamento della storia naturale e geografia	1917
Cardboards for geometric developments of wooden models	cardboard			YES	NO	YES	67	L.4				
Collection of 50 pieces	cardboard	50	70x100 mm	YES	NO	NO	66	L.140				
Collection of 15 pieces. Main forms of all systems: constructed with glass sheets; including the axes and the basic cardboard shape	glass	15		NO	NO	YES	67	L.110				
Collection of 29 models		29		NO	NO	NO	67	L.210				
Collection of wooden crystal models. Series of 52 models of the main simple and compound shapes, in a box. Series of 52 models, in a box.	wood	52		NO	YES	YES	85	L.140	Mineralogy, Geology and Paleontology	Crystallographic models and natural crystals	Catalogo dei sussidi didattici per l'insegnamento delle scienze naturali	1932
Collection of 53 large format cardboard models. Systems: monometric, tetragonal, hexagonal and rhombohedral, rhombic and trimetric, monoclinic, triclinic	cardboard	53 (52 (forms from the wooden collection) +1 (dodecagon bipyramid))	The edge of the cube is approximately 3 times that of the wooden models.	YES	NO	NO	85	L.400				
Collection of 15 glass crystal models with colored silk thread axes	glass	15		YES	NO	YES	85, 86	L.340				
Collection of glass crystal models with colored silk thread axes, consisting of 19 shapes	glass	19 (15 forms from the glass collection + 4 (Tetrakishexahedron, icositetrahedron, hexacisoctahedron, dodecahedron pentagon with tetrakishexahedron included (in cardboard))		YES	NO	YES	86	L.490				
Collection of 25 glass crystals, with internal axes represented by stretched threads of different color for the different axes, or with cardboard holohedral shapes inside (the Collection cannot be subdivided)	glass	25		NO	NO	NO	87	L.1150				
Wooden crystals, models representing the main simple and compound forms. Series of 52 models in a box	wood	52		NO	YES	YES	155	L.175	Mineralogy, Metallography, Crystallography, Geology, Paleontology	Crystallographic models	Sussidi didattici per le scienze naturali e l'agraria	1941
Wooden crystals. Series of 22 models	wood	22		NO	NO	NO	155	L.130				
Collection of 53 cardboard models, large format	cardboard	53 (forms from the collection of 52 wooden models + dodecagonal bipyramid)	The edge of the cube is approximately 3 times that of the wooden models	YES	NO	NO	156	L.500				
Collection of 53 cardboard models, large format with red edges for greater emphasis on the crystal	cardboard	53 (forms from the collection of 52 wooden models + dodecagonal bipyramid)	The edge of the cube is approximately 3 times that of the wooden models	YES	NO	NO	156	L.590				
Reduced collection of 21 cardboard models, large format, representing the main shapes of the 6 crystalline systems	cardboard	21		YES	NO	NO	156	L.265				
Reduced collection of 21 cardboard models, large format, representing the main shapes of the 6 crystalline systems with the edges in red to give greater emphasis to the crystal	cardboard	21		YES	NO	NO	156	L.305				
Collection of 15 glass crystal models with colored silk thread axes	glass	15		YES	NO	YES	157	L.420				
Collection of glass crystal models, 19 shapes with colored silk thread axes	glass	19 (15 forms from the glass collection + 4 (Tetrakishexahedron, icositetrahedron, hexacisoctahedron, pentagon dodecahedron with tetrakishexahedron included (in cardboard))		YES	NO	YES	157	L.630				
Series of 52 crystallographic wooden shapes, including crystals of the main systems, with simple and compound shapes. The series is enclosed in a double wooden box	wood	52		NO	YES	YES	10		Natural Science	Crystallography, Mineralogy, Geology	Scienze naturali, fisica, chimica. Collezioni e Gabinetti. Listino prezzi	1942
Series of 19 glass crystallographic forms with internal axes in colored silk thread	vetro	19		NO	NO	SI	17, 39					
Collection of 52 wooden crystal models, representing the main simple and compound shapes. In wooden box	wood	52		NO	YES	YES	84		Mineralogy, Metallography, Crystallography, Geology, Paleontology	Crystallographic models	Materiali didattici. Parte seconda. I Cataloghi della rinascita	1951
Collection of 22 wooden crystal models, representing the main simple and compound shapes.	wood	22		NO	NO	NO	84					
Collection of 19 glass crystal models with colored silk thread axes	glass	19	Cube dimensions: 10 cm on the side; the other shapes in proportion	YES	NO	YES	84					
Collection of 52 wooden crystal models, representing the main simple and compound shapes. In a wooden box with two shelves, with divisions inside.	wood	52		NO	YES	NO	79	35.200	Mineralogy and Crystallography	Crystallographic models	Scienze naturali, agraria, tecno-merceologia, chimica	1958
Collection of 22 wooden crystal models, in a box with sliding lid and internal divisions.	wood	22		NO	YES	NO	79	15.150				
Collection of 27 glass crystal models with colored silk thread axes or hemihedral shapes included.	glass	27	Cube dimensions: approx. 12 cm on the side; the other shapes in proportion	YES	NO	YES	79	on demand				
Wooden crystals: series of 52 models in a box, representing the main shapes, simple and compound.	wood	52		NO	YES	YES	57		Geology, Crystallography, Mineralogy	Crystallographic models	Materiali sussidi didattici e scientifici	1959
Crystals, in wood: series of 22 models in a box	wood	22		NO	YES	NO	57					
Collection of 52 wooden crystal models representing the main simple and compound shapes. In a wooden box with two shelves, with divisions inside.	wood	52		NO	YES	YES	79		Mineralogy and Crystallography	Crystallographic models	Scienze naturali, agraria, tecno-merceologia, chimica	1968
Collection of 22 models of wooden crystals, in a box with sliding lid and internal divisions.	wood	22		NO	YES	NO	79					
Collection of 27 glass crystal models with colored silk thread axes or hemihedral shapes included	glass	27	Cube dimensions: approx. 12 cm on the side; the other shapes in proportion	YES	NO	YES	79					
Collection of 52 wooden crystal models, representing the main simple and compound shapes. In a wooden box with two shelves, with divisions inside	wood	52		NO	YES	NO	146		Science of the Earth and the Universe	Mineralogy and Crystallography	Scienze naturali Anatomia umana, zoologia, botanica, geografia, agraria, tecno-merceologia	1980
Collection of 22 models of wooden crystals, in a box with sliding lid and internal divisions	wood	22		NO	YES	NO	146					
									Science of the Earth and the Universe	Mineralogy and Crystallography	Chimica, biologia, scienze della terra	1987

Fig. 12. Crystallographic forms' models in the Catalogs of material and educational aids for the teaching of Natural Sciences, G.B. Paravia & C., editions from 1917 to 1987. Table by P. Argenziano, A. Avella, N. Pisacane, 2024.

Fig. 13. Catalogs of educational aids for the teaching of Natural Sciences, G.B. Paravia & C., Covers, editions of 1917, 1932, 1941, 1942, 1951 (top from left to right), 1956, 1959, 1968, 1980, 1987 (bottom from left to right).

The comparative analysis shows that the 15 glass specimens on display at the Michelangelo Museum do not belong to the complete series of 15 models of crystallographic forms in the 1917 and 1932 catalogs (Paravia & C., 1917, 1932) missing from these series the cube, the rhombododecahedron, the triacisoctahedron, the tetrahedron with the octahedron included (cardboard), the square-based prism and the monoclinic octahedron. They could be part of the collection of 29 models proposed in the 1917 catalog, in which a specific description is not given. The Museum's specimens cannot even belong to the complete series of 19 models in the catalog since 1932 which integrates the series of 15 models with the forms of the tetracisesahedron, the hexacisoctahedron, the icositetrahedron and the pentagon-dodecahedron with the tetracisesahedron included (in cardboard) because of these four forms only the last two are present among the Museum's specimens. The models could be part of a larger series with 25 pieces also available in the 1932 catalog. The six pieces that integrate this series are not described.

The forms of the pentagon-dodecahedron with axes, the rhombohedron with axes and the monoclinic prism present among the models on display are listed and described for the first time in the collection of 27 glass models proposed in the 1956 catalog and illustrated for the first time with a black and white drawing in Plate no. 73 attached to the 1968 catalog (Paravia & C., 1956, 1968) **(Fig. 14)**.

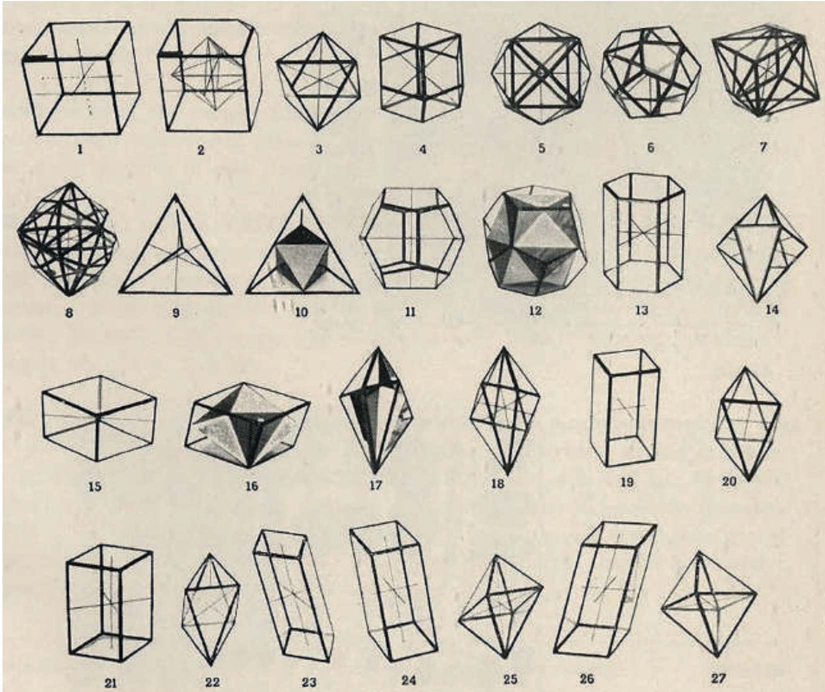


Fig. 14. Paravia, Complete collection of 27 crystallographic forms' glass models. (Paravia & C., 1968)

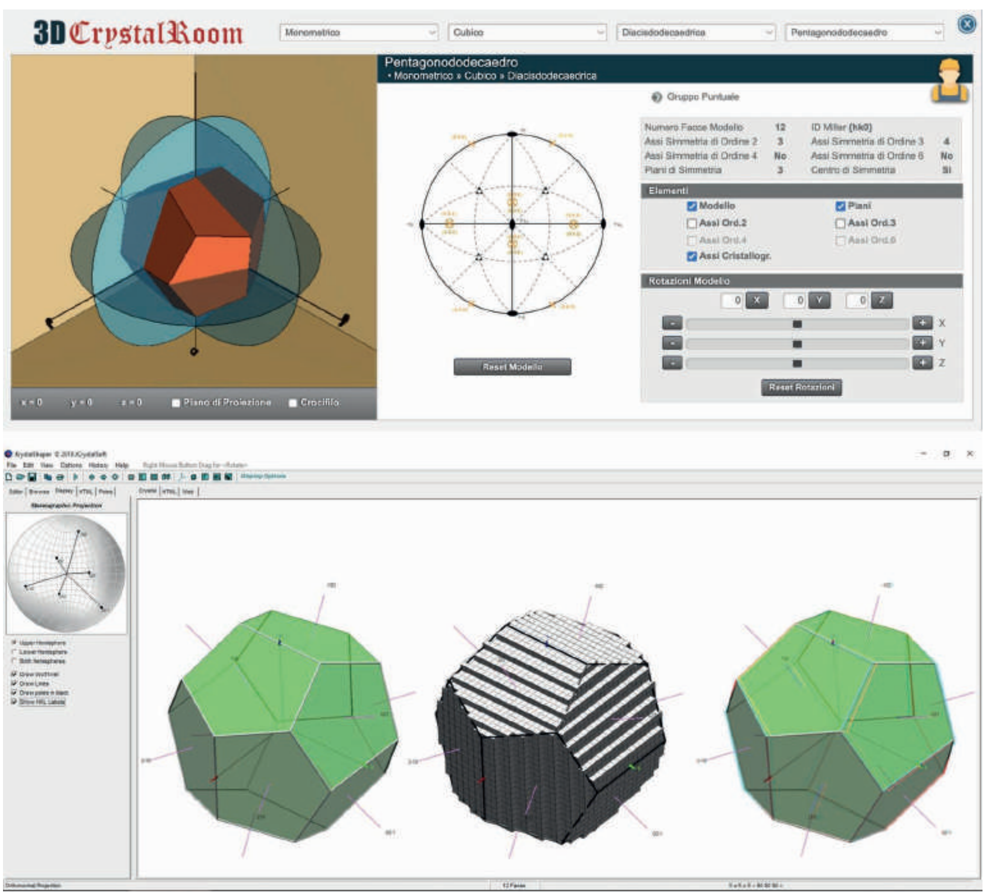
It can therefore be deduced that the 15 specimens on display are part of a series with 25 or more pieces that may be one of those proposed in the catalogs mentioned above.

After the systematization of the fundamental crystallographic forms and the underlying geometric relationships through comparative analysis with analog models, the study continued by observing the three-dimensional models of crystallographic structures also in a digital environment, through databases and specific mineralogy software[12]. Thanks to the advancement of digital technologies in the crystallographic and architectural fields, the digital representation of models allows the facilitated and intuitive observation and understanding of the specificities of crystal forms consolidating also in digital environment the critical knowledge of complex solids and of the main geometric characteristics, as was the teaching intention of Prof. K. Vrba of the University of Prague at the end of the nineteenth century, taken up and expanded by F. Krantz. The graphic interfaces of the various software used present the crystals in dynamic axonometry and sometimes in the 'block' representation by Haüy (1743-1822) and in anaglyphs. Through the three representations, it is possible to visualize the axes, in the various orders, and the planes of symmetry, that are the fundamental geometric characteristics of solids. In the same software, the pseudo-three-dimensional representations are flanked by the crystal stereogram, which is the most

12 Among the experimented software are: JCrystal, KrystalShaper, WinWULFF and 3DCrystalRoom of the University of Bari Aldo Moro

complete plane representation of the solid and its elements of symmetry through a symbolic encoding, valid regardless of its regular conformation or not. Stereographic projection has been applied to the crystals precisely to represent the edges and vertices of the faces and their symmetry relationships; one of the first to propose this representation was Bravais (see Chapter 3). The crystal lattice is positioned at the central point of the sphere and the crystallographic directions (edges and vertices) are projected onto the surface of the sphere, from the opposite poles of the vertical main axis.

The digital models and data collections made available by whether free or open-source software allow to interactively deepen the specificities of crystallographic forms based on cataloging by group, system and class or indexing by William H. Miller (1801-1880), still in use in the crystallographic field, with obvious advantages in the visualization of crystallographic forms different from those already stored in the software database. With reference to the triple cataloging, moreover, it was possible to model "new" crystals by modifying the Miller indices, by varying the orthogonal direction to a specific crystal lattice plane, determined in the vector environmental. **(Fig. 15)**



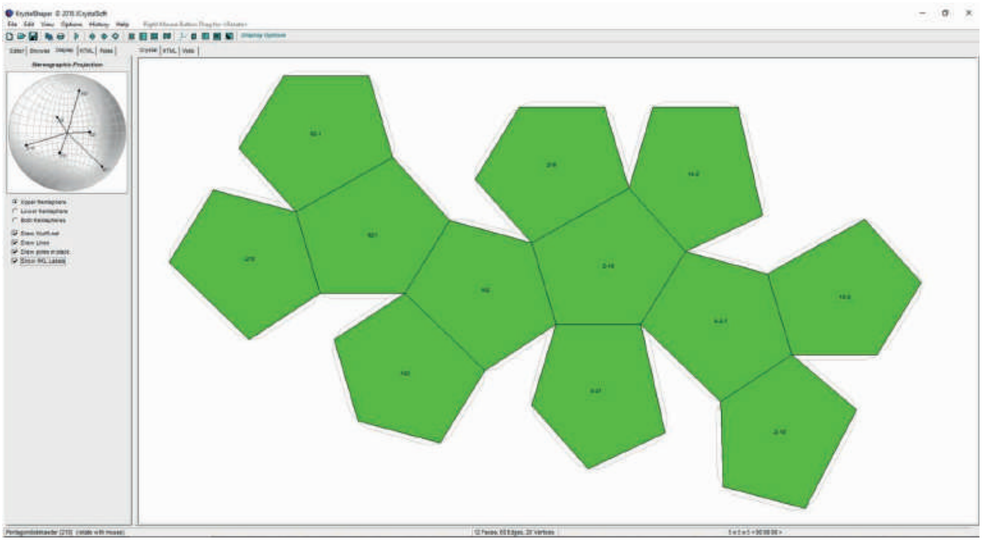


Fig. 15. The crystal form of the pentagon-dodecahedron. Digital model and stereogram processed in the 3DCrystalRoom software (University of Bari) (top). Stereogram, digital model, block model, anaglyphs (middle), and stereogram and net of pentagon-dodecahedron (bottom), processed in the KrystalShaper software. Digital elaboration by P. Argenziano, A. Avella, and N. Pisacane, 2024.

DIGITAL CRYSTALLOGRAPHY, FROM 3D SURVEYING TO PARAMETRIC MODELING

Suppose a first scientific systematization of crystallographic forms dates back to the second half of the Eighteenth century by the French mineralogist and crystallographer René Just Haüy. In that case, the scientific dissertation of crystallography has older origins that can be traced back to the *Naturalis Historia* by Gaius Plinius Secundus (named Pliny the Elder), (77-78 AD) (Plinius Secundus, 1489). The Latin writer dedicates the concluding books of his encyclopaedic work on the Natural Sciences to stones, examined as materials present in Nature regardless of their origin and their use through man's creation of statues, architectural monuments, and gems engraved or set in jewels. Starting from natural stones in general, the author devotes the last five books to mineralogy in general, focusing on its uses in architecture in Book XXXVI and on precious gems and semiprecious stones in Book XXXVII. The work even of great interest does not report pictures or drawings.

The assumptions by Pliny the Elder about the common natural origin of solid materials allow us to deduce the principles of precious stone processing from the disciplines that study the processing of building stones. Regardless of the type of material being machined and the scope of application, disciplines that study the processing of solid materials recognize their common foundation in Geometry.

As its indicates, the Stereotomy, developed in the broader of Descriptive Geometry, is generally aimed at cutting isotropic materials (stones, wood, metals) in the context of buildings construction. Its widest application concerns stone materials in architecture.

It is no coincidence that this discipline, which had wide diffusion in Europe, found a fertile background in France due to the extensive use of calcareous and sedimentary rocks for the construction ~~of~~ buildings and infrastructures (Pérouse de Montclos, 2001).

In direct link with Stereotomy, when isotropic materials are natural mineral substances, and the field of application refers to the manufacturing of precious stones, the discipline is 'lapidary art'[13] that still recognises its rules in geometry.

As in architecture, Stereotomy bases its principles on geometric rules for processing rock materials, starting from their specific characteristics, to ensure the aesthetics and stability of the buildings. In gemology, lapidary art uses geometry to orient the cut of precious stones to obtain a greater crystal brilliance with the least dispersion of precious material.

Since the Twentieth century, in the field of lapidary techniques applied to precious stones, specific studies on the determination of input parameters for the design of models, including digital models in specific parametric modeling software, have been undertaken and are still ongoing.

Extending Riccardo Migliari's thought from architectural models to crystallographic ones, models - whether digital or tangible – allow to reconstruct in the 3D space the geometric model of the represented object. (Migliari, 2003)

In architecture, the task of tangible models to realistically anticipate the built environment is nowadays delegated almost exclusively to digital representation, in which the model is thus represented more abstractly, investing it with the possibility of enunciating specific aspects of the project. In the field of gemology, using computer models to design the cuts applied to precious stones makes it possible to control the design workflow and *ex ante* to evaluate the result in terms of maximizing the optical characteristics of the precious stones and the yield obtained.

In the last twenty years parametric modeling software have been developed in the field of gemology. The best known are "GemCad – GemRay"[14] and "Gem Cut Studio"[15]. As the geometry of the cut stone is parametric and in direct link with crystallography, the translation between analog and digital processes has been easier in gemology than in the different fields of AEC.

13 In lapidary art, the minerals are classified into precious stone, semi-precious stone or gem, and pietra dura according to characteristics of decreasing transparency (precious stone has high levels of transparency and hard stone has no prevailing transparency) and different morphological, physical and chemical properties. The lapidary art in the manufacturing of minerals for ornamental purposes dates back, according to historians, to the times of the river civilisations of the Mesopotamian area (V century of BC), where the polishing of the genuine faces of crystals and of pebbles belonging to alluvial deposits was made. Only later, the invention of cabochon manufacturing introduced the polishing of dome minerals without cuts, with smooth and rounded edges. The first lapidary models cut into facets to obtain prismatic shapes that enhanced the known qualities of brilliance, started from the XIII century AC. The most complex processing systems, such as carving and engraving, and refining originated from simple polishing and crystal faceting processing systems. All these processes belong to the artistic technique called "glyptic" (from the Greek "carving") and were transmitted almost unchanged until the introduction of mechanisation processes in the XVIII century. (Nicols, 1652; Sborgi, 1973)

14 (*GemCad - GemRay*, n.d.)

15 (*Gem Cut Studio*, n.d.)

The digital approach follows the traditional one in which faceting tools are basically goniometers. The raw stone is put in the center of this tool and rotates about the "gear indexes". The latter are set according to the periodicity of the cutting angles and the planes of symmetry, which depend on the specific mineralogical characteristics of the stone. The gear index is a numerical value corresponding to the equal division of the turn angle, which is countable in: 32, 64, 72, 77, 80, 84, 86, 96 and 120. Among these, the toothed crowns with 64 and 96 gear indices allow the greatest number of cuts in application to the greatest number of stones; conversely, the toothed crown 77 with gear index is used only for three types of cutting.

The number and angle of the cutting planes are the basis of the parametric modelling software made for the digital design of precious stones. Digital modelling starts from a primitive solid (cylinder or cube depending on the software) (Sangveraphunsiri et al., 2018) in whose center of mass a tris-orthogonal triplet is fixed, in analogy to crystallography; the degree of symmetry of the model to be realised, the cleavage angles, and the gear indices are set with respect to same triplet, in analogy to the faceting procedures.

The digital model of the cut stone thus obtained can then be subjected to the analytical verification of brilliance[16] through photorealistic rendering, the simulation of light trajectories within the gem and analytical diagrams (tilt performance); three possible operations thanks to the mineralogical characteristics of the stone previously set in the software with advantages in the faceting design. From the three-dimensional model of the cut stone, a technical sheet is automatically produced that shows the cut's geometric characteristics and the solid's representation in quadruple orthogonal projection **(Fig. 16)**.

16 The two activities of faceting and analytical verification of the brilliance of the cut-stone are carried out by exchanging files in the two distinct software "Gem Cad" and "GemRay", designed by Robert W. Strickland in 2002; while in "Gem Cut Studio", designed by Rej Poirier in 2022, the two activities are carried out in real time in a single software with obvious advantages in the project workflow.

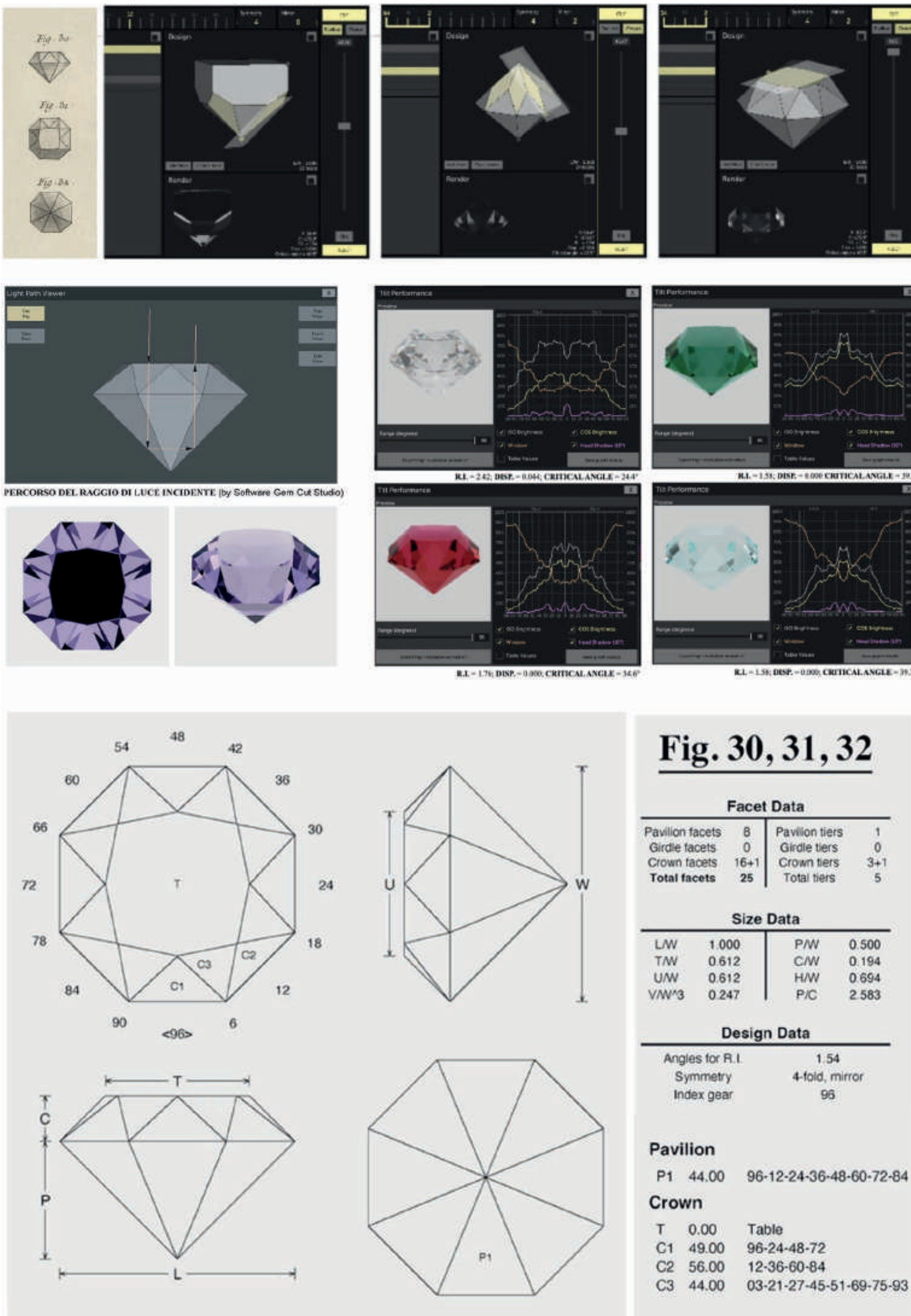


Fig. 16. Parametric 3D modeling in “Gem Cut Studio” of the gem drawing 30, 31, 32 in the Plate II Orfèvre Jouaillier, Metteur en Oeuvre, Taille des Diamans by Denis Diderot. Drawing by P. Argenziano, A. Avella, and N. Pisacane, 2023.

In gemology, improving precious stones' analog or digital processing tools corresponds to the technological advancement of surveying tools. This demonstrates a known theoretical aspect both in the lapidary field and in other fields of technique applied to the arts: survey and design activities are mutually linked through geometry and measurement, about the material characteristics of the stone. The material determines its form, the geometry presides over its conformation and is fundamental for its study; the measurement defines the dimensional scalarities of the object – about the S.I. – within standard values of tolerance and accuracy.

Surveying a rough stone means characterising its mineralogical nature in terms of planes of cleavage, the class, and the degree of symmetry of the stone. According to this, it is essential to design the best possible cut, looking for the right balance between the values of brilliance and the waste of material; if there were no such balance, the brilliance would decay after faceting, or even the cutting process would not be successful.

On the other hand, surveying a cut stone means determining: its form by measuring the width of the solid angles between the facets; its overall dimensions in its articulation into crown, belt, table, pavilion; and its particular ones by measuring the length of the edges of the facets; in other words, surveying a cut stone means analytically estimating the quality of the artistry concerning the cutting project, and in some cases (such as the diamond) identifying a specific precious stone among numerous similar ones. (GIA Researchteam, 2005)

The experience of gemstone surveying can be applied to mounted precious stones, which can be appreciated in numerous collections. This opens up critical comparisons among the jewelry modelling, contemporary iconography (in works of art and treatises) and processing technology. In this context, digital modelling of the jewel can strengthen the investigative experience by redesigning iconographic sources or more recent patents.

The development of two-circle goniometers for faceting corresponded in the Nineteenth century to the development of two-circle reflecting goniometer to measure inter-facet angles of the stones and then evaluate the internal reflection of light. These tools were used until the invention of new one based on the X-ray diffraction technique; in some cases, the former still attest to a high degree of accuracy of measurements. (Shen et al., 2012)

In the geometric survey of the cut-stone, the caliber and the precision balance are still the main tool for measurements and weight of the raw or cut stone. In particular, the caliber given its high standard of metric accuracy about to the instrumental type. The most used calibers are three with accuracies of ±0.05 mm, ±0.01 mm and ±0.001 mm. The precision balance to measure the weight of the stone (raw or cut) expressed in carats that are as decisive as the brilliance for a complete characterization.

Digitisation has also been applied to the field of gemology. Therefore, close-range digital photogrammetry and 3D laser scanner are being tested for surveying and modelling cut-stone; the photogrammetric model becomes the "digital twin" of the real cut stone, and so geometric and metric evaluations can be carried out into the digital environment.

The digital twin of the cut-stone, as in the architectural field for the Building Information Modeling, becomes the geometric database of information modeling, which is also extended to jewelry. This is useful both for constructing digital archives for expanded use of the same models, and for the association of information databases that can always be implemented to support design and evaluation processes.

THE SCIENTIFIC REPRESENTATION OF CRYSTAL POLYHEDRA. INSIGHTS AND METHODOLOGICAL INNOVATIONS (XVII-XX CENTURIES)

by Pasquale Argenziano

In mineralogy and crystallography, tangible modeling is not long past scientific matter – see Chapter 2 – compared to the number of texts that have dealt with the natural sciences since ancient times, in various capacities, and which were sometimes enriched with drawings from life to illustrate mining landscapes, rocks and crystals. The authors accurately described natural phenomena, entrusting the artists with the representation of every subject, inspired more by verisimilitude than by graphic documentation (Plinio Secondo, 1988).

According to their most recent meanings, jewelry and gemology have the same fate as the abovementioned sciences. The full-size modeling coincides with the first purpose of jewelry; Drawing is evidently the medium to fix the design thought, and to make it evolve, then the craftsmanship is directly related to the mastery of goldsmiths and carvers. So much so that the drawings of jewels, in archives or published, are few compared to the number of masterpieces we can admire in museums or private collections. The regularization of patent dates to the end of the Nineteenth Century; Therefore, only from that time, the quantity and quality of the jewelry drawings, precious metal mountings, and faceted gems increased considerably[1]. Until then, most of the representations of jewels can be traced in figure paintings of the nobility and wealthy who attested to their social status through clothing and jewelry (De Fiore, 1983).

The transition from the symbolic representation of minerals and jewelry to the technical one develops in the cultural framework of the French Enlightenment. The first drawings of crystals, jewels and gems – made by methods and techniques close to those currently used – are published in the tables' volumes of the *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers* (1762-1772) by Diderot and D'Alembert. And again, the scientific representation of crystals, understood as polyhedra, by a method that controls the projection of solids on the plane scientifically, and that document their geometric characteristics between reality and image – the Parallel Projections – was first introduced by Romé de L'Isle and then systematized by Haüy, similarly to what happened for tangible models, described in the previous chapter.

1 The *Paris Convention for the Protection of Industrial Property*, signed on 20 March 1883, was among the first international treaties on intellectual and industrial property; In particular, Article 4 regulates for the first time the use of drawings and of tangible models (mockups) for the copyright on projects and intellectual works.

In Germany, the studies of crystallography and geometry intertwined Haüy's early research with the developments of spherical trigonometry. They led to the definition of the stereographic method in the representation of crystals, directly derived from contemporary cartographic applications.

THE SCIENTIFIC DRAWING OF CRYSTALS AND GEMS IN THE FRENCH POLYTECHNIC CULTURE

Jean-Baptiste Louis Romé de L'Isle (1736-1790) laid the scientific basis of mineralogy and crystallography through two books: *Essai de cristallographie, ou description des figures géométriques propres à différens corps du regne minéral, connus vulgairement sous le nom de cristaux* (Paris 1772), and *Cristallographie, ou description des formes propres a tous les corps du regne minéral* (Paris 1783, in four volumes), both illustrated by drawings supporting the theoretical concepts and summaries of the most important data.

In the first essay, Rome de L'Isle consciously adopts and makes his own the classificatory and graphic method (Romé de L'Isle, 1772) proposed by the naturalist Carl von Linné (Linné (von), 1766) [2] that:

> «[…] est jusqu'à présent le seul qui soit entré dans ces détails, & qui au moyen des figures & des développemens qu'il a donné des Cristaux qu'il connoissoit, ait mis tous les Naturalistes à portée de les reconnoître. C'est d'après l'examen réfléchi de la figure totale & partielle de chacun de ces Cristaux que ce célebre Naturaliste a cru trouver entre plusieurs d'entr'eux une analogie assez décidée pour les rapports à certains Genres, dans fa Table des Affinités Cristaux; […]»

In ten plates [3] attached to his first book (1772), Rome de L'Isle adopts the net of crystal polyhedra – that is, the method of representation that von Linné experimented on crystals **(Figg. 1, 2)** – juxtaposing, in the same table, the net of polyhedra with more complex crystal forms **(Figg. 3, 4)**. The geometrical data, recurring in the net of the twenty-five crystals examined, are then summarized in the tenth plate. In the *Tableau Cristallographique*, which precedes the plates, there are analytical data on mineralogy, morphology and geometry, which are examples of the text and necessary for understanding the drawings.

2 Carl von Linné devotes the chapter *Systemata lapidum, sequentia praecipua sunt* to minerals and crystals; The drawings are organized in three plates: in the first one, there are the chiaroscuro drawings of the crystals, in the following ones the net of crystal polyhedra.
3 The ten plates were drawn and engraved by Bresse.

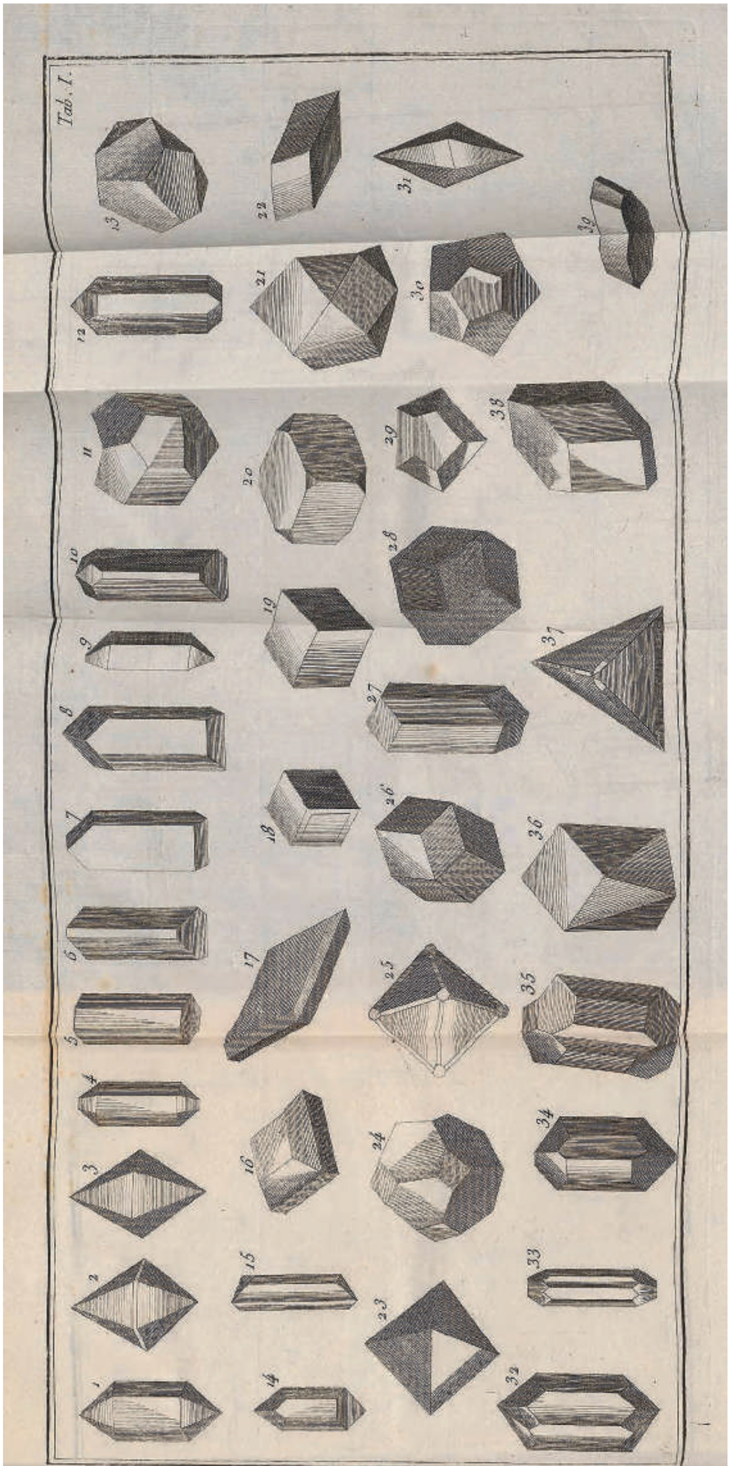


Fig. 1. Carl von Linné, Pseudo-axonometric chiaroscuros drawings of 39 crystals. (von Linné, 1766, plate I)

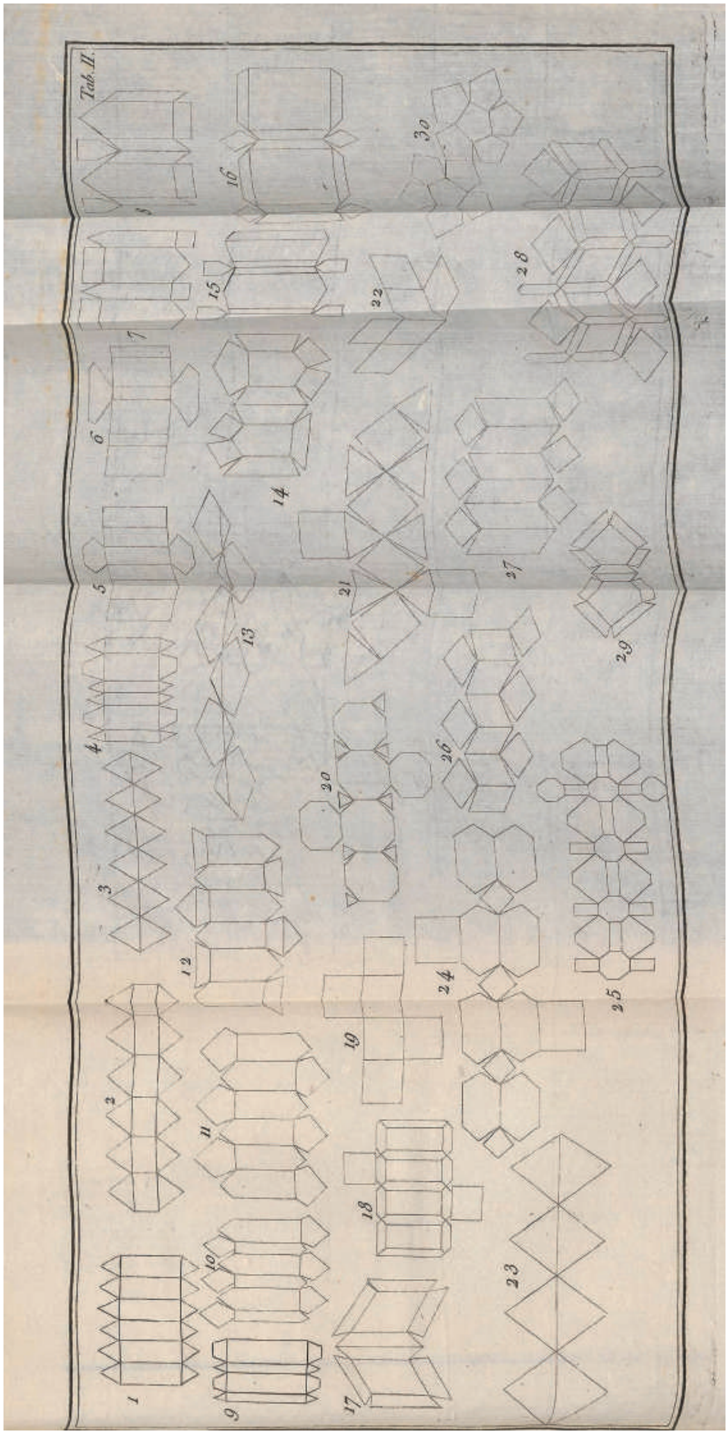


Fig. 2. Carl von Linné, The net of some crystal polyhedra drawn in plate I. (von Linné, 1766, plate II)

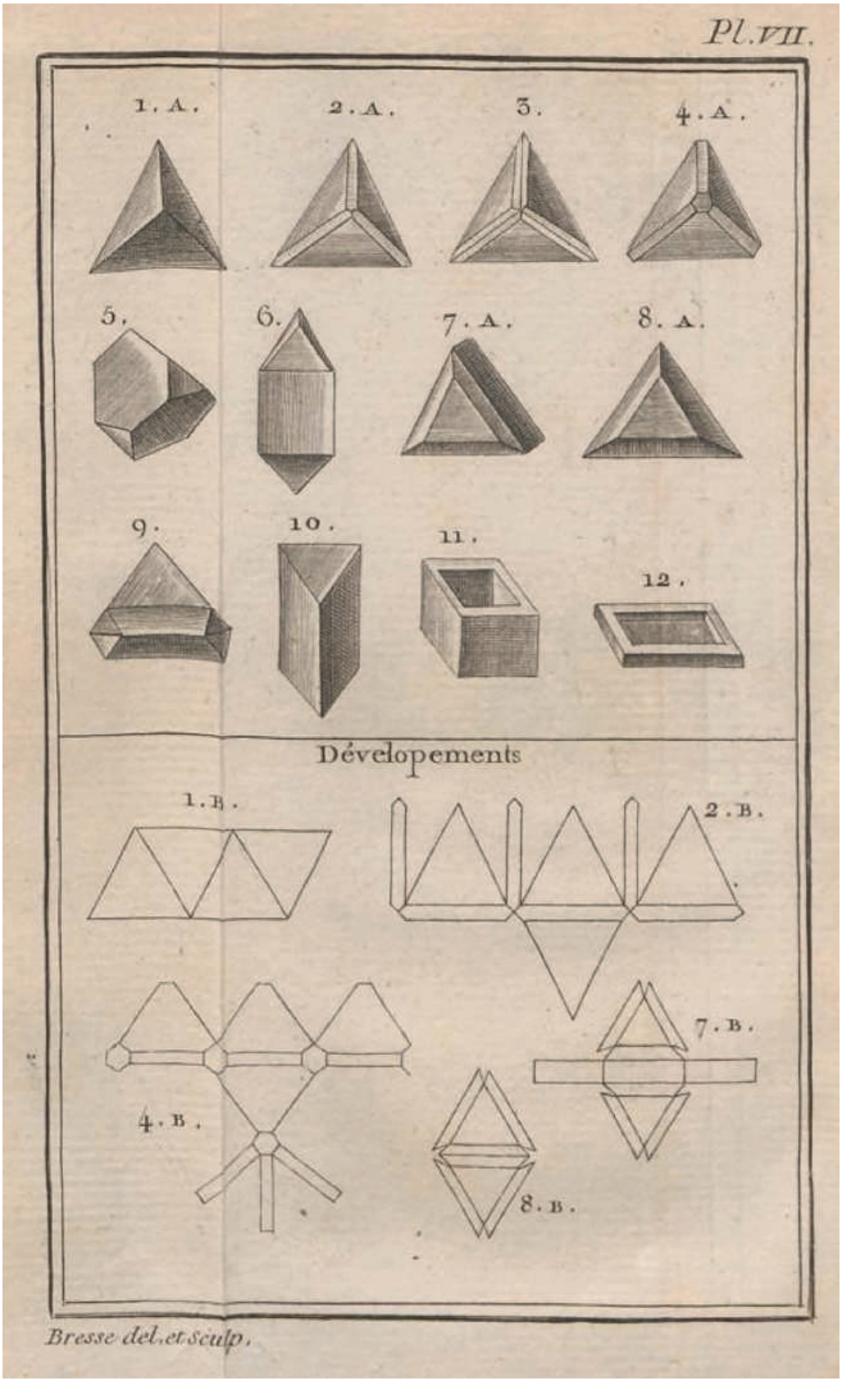


Fig. 3. Jean-Baptiste Louis Romé de L'Isle, Crystal polyhedra with triangular matrix, simple and composite: the comparison among the pseudo-axonometric drawings and their net. (Romé de L'Isle, 1772, plate VII)

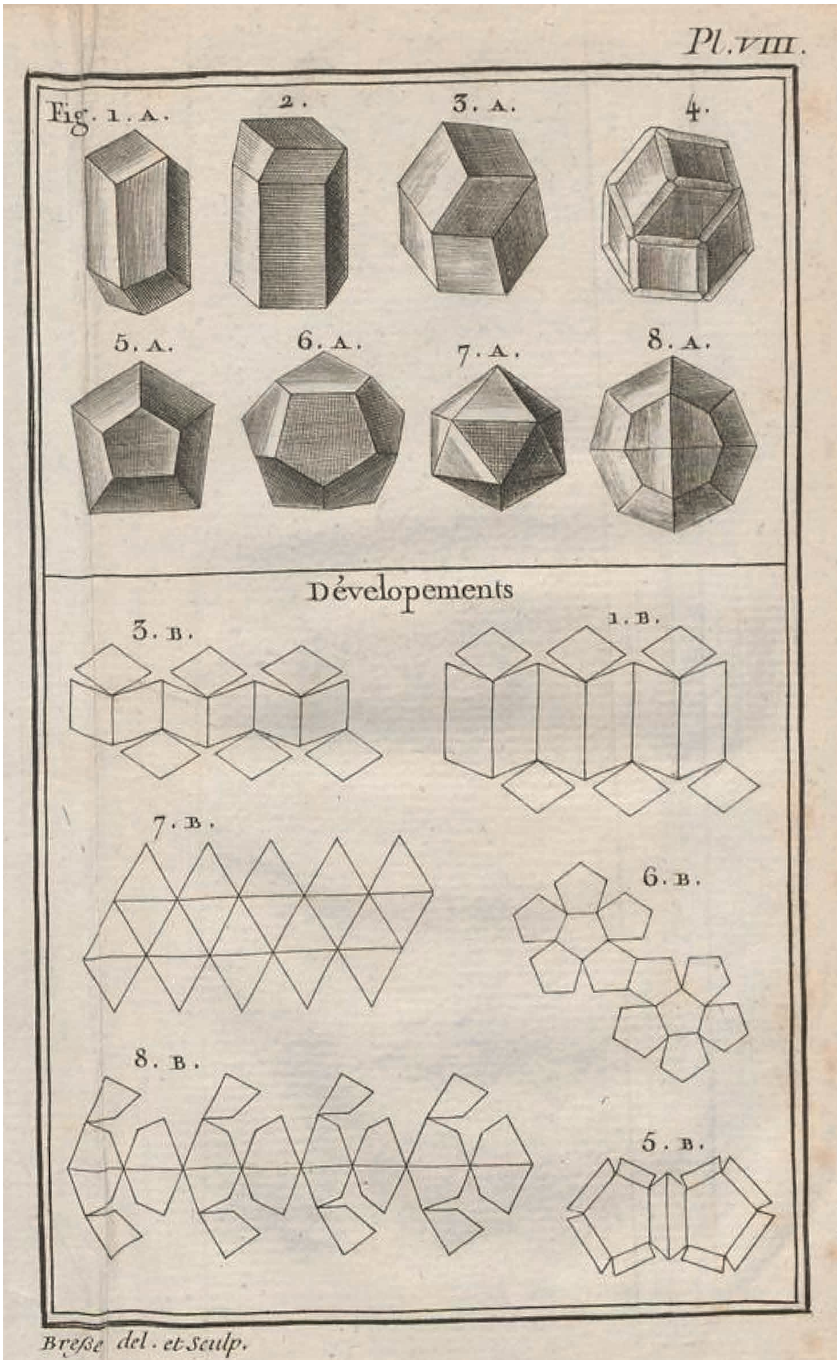


Fig. 4. Jean-Baptiste Louis Romé de L'Isle, Crystal polyhedra with triangular, quadrangular, pentagonal matrix: the comparison among the pseudo-axonometric drawings and their net. I(Romé de L'Isle, 1772, plate VII)

In the second book (1783), Rome de L'Isle improved the criteria of investigation and abandoned the net of crystal polyhedra in favor of pseudo-axonometric representations, the first method Carl von Linné chose in his book. The French scientist classified crystals starting from the primitive forms, and organized the polyhedra drawings in 12 plates, each accompanied by one or more in-depth tables, described in the first chapter of this book. This choice is linked to the work of tangible crystal models and the measurements able by the "contact goniometer" – as written in the Preface of that book and then described by Miller (Miller, 1839; Romé de L'Isle, 1783) – invented by Arnould Carangeot (1742-1806) in 1780. **(fig. 5)**

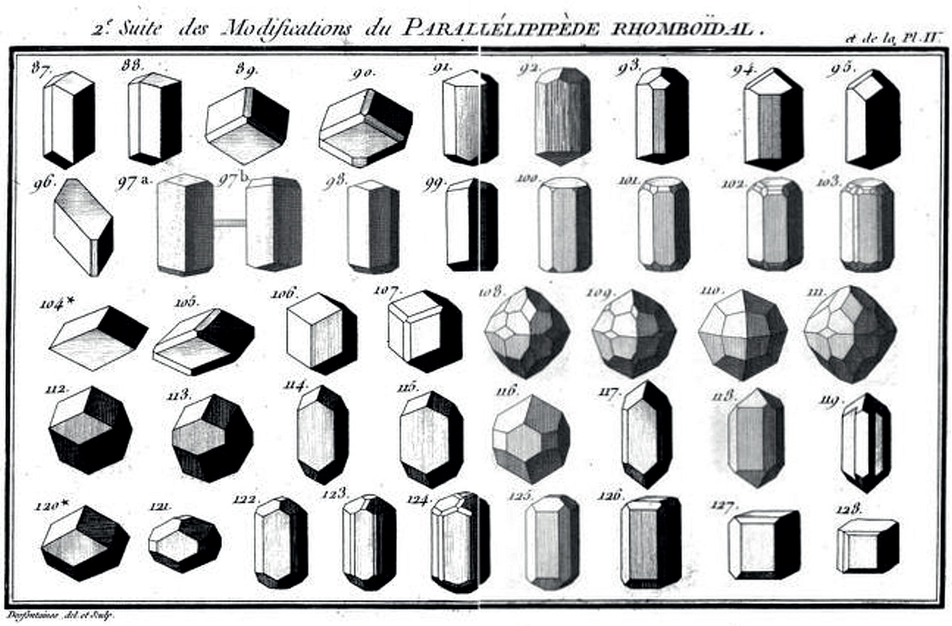


Fig. 5. Jean-Baptiste Louis Romé de L'Isle, The crystal polyhedra of the class "Rhomboidal parallelepiped". (Romé de L'Isle, 1783, plate IV)

The research of the two French scientists intertwined, as written in their books.

René-Just Haüy (1743-1822) publishes the first of his four works on crystallography a year after the last book by Rome de L'Isle: the *Essai d'une théorie sur la structure des crystaux. Appliquée à plusieurs genres de substances crystallisées* (Paris 1784); every twenty years, he published the developments of his research in the *Traité de minéralogie* (1801, in five volumes) and in the *Traité de cristallographie* (1822, in three volumes).

Among these two very important works, the French scientist published *Traité des caractères physiques des pierres précieuses , pour servir à leur détermination, lorsqu'elles ont été taillées* (Paris 1817) dedicating a specific study «sur les substances qui fournissent aux artistes la matière des objets d'agrément que l'on désigne sous le nom de pierres précieuses», unlike his precursors Johan Gottschalk Wallerius (1709–1785) and J.B.L. Romé de L'Isle (Haüy, 1817, pp. i–ii).

In forty years, Haüy gradually developed a systematic and analytical approach minerals and crystals, defining the geometric rules underlying their conformation, the methods of analysis, and the classification criteria still largely used in crystallography today. Furthermore, Haüy defines for the first time the method and techniques of crystal drawing to support the theoretical concepts, and the documentation downstream of rigorous measurements on specimens of various sizes. This aspect cannot be overlooked if we consider that Romé de L'Isle proposed two different graphic approaches to the representation of crystals – as described above – the first extremely unreal (the net of polyhedra) and the second one figurative.

Haüy – as main superintendent of the *École de mines de Paris* collections – had access to one of the most complete mineralogical collections of his time, and to the most suitable laboratories and instruments for their systematic study, defining a key to technical-scientific analysis that was innovative compared to its precursors. It is an approach based on optics, physics and chemistry of materials and, simultaneously, on the methods, techniques and instruments of measurement and graphic representation [4], characteristic of the French polytechnic culture.

As discussed below, Haüy most likely had bibliographical – if not direct – contacts with the *Encyclopedie* environment and Gaspard Monge. Nor can it be ruled out that Haüy had news of William Farish's (1759-1837) teaching activities at Cambridge in the field of arts and manufactures and chemistry (including metals and minerals) and of his research on the machines that led him to coin the method of isometric axonometry (Càndito, 2003; Farish, 1796, 1820; Giordano, 2001; Loria, 1921; Scolari, 1984).

The analysis of the plates attached to Haüy's three books[5] in comparison with their texts has shown an evolution in theoretical and applied awareness, representation methods, and graphic techniques. **(Fig. 6)**

4 In the introductions to his books, Haüy makes bibliographical reference to numerous authors, first Torbern Bergman (1735-1784) who published some pseudo-axonometric drawings in (Bergman, 1792).

5 The plates in the 1784 book were drawn by Fossier and engraved by Selier. The plates in the 1801 book were drawn and engraved by Maleuvre.

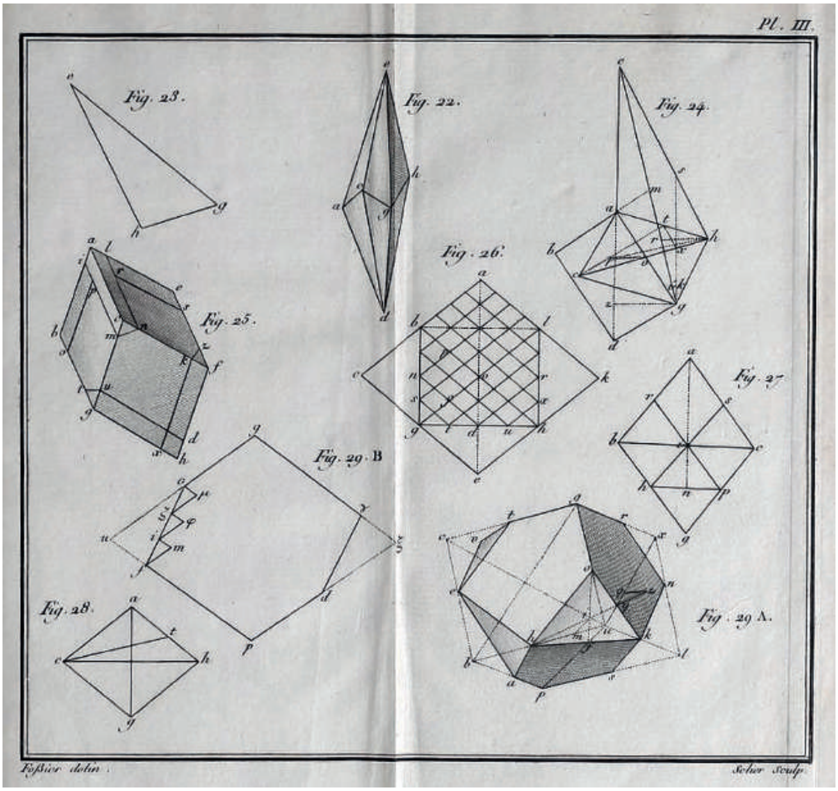


Fig. 6. René-Just Haüy, Volumetric and plane analysis of the crystals of “Spath calcaire rhomboïdal a sommets Aigus”. (Haüy, 1784, pp.108-16, plate III)

While the axonometries of the crystals and the projection of their plane faces are used without a specific scientific explanation in the eight plates attached to the 1784 book, Haüy clarifies in the Treatise of 1801: «[…] Les figures ont été tracées d’après la méthode des projections, en supposant le point de vue éloigné à l’infini. Les lignes pleines représentent les arêtes situées dans la partie du solide qui seroit tournée vers l’observateur, s’il le voyoit dans la position à laquelle se rapporte la projection; et les lignes ponctuées représentent les arêtes situées dans la partie oppose, ou celle que l’observateur ne pourroit apercevoir, qu’en supposant le solide diaphane. Dans les figures relatives aux constructions géométriques, on a représenté les diagonales et autres lignes couchées sur les faces du solide, par des suites de lignes partielles, qui laissent entre elles de petits vides; voyez mr, cm, cr (fig. 4), pl. IX; et bg, ad, bf, fg, fs (fig. 9), ibid; et l’on a représenté les axes et autres lignes qui traversent le solide, ainsi que celles qui sont extérieures à son égard, par des suites de lignes partielles, avec des points intermédiaires. Voyez cg (fig. 9) pl. IX, et MR, CM, CR (fig. 4), ibid. On pourra remarquer sur cette même figure, que les parties supérieures des lignes Ms, Ru, qui se trouvent situées dans l’espace, sont des assemblages de lignes partielles entremêlées de points, tandis que leurs parties inférieures, qui s’appliquent sur la surface du solide, sont composées de lignes partielles sans points intermédiaires. Cette distribution, dont l'idée heureuse est due au Cit. Tremery, ingénieur des mines, aidera le lecteur à se reconnoitre dans l'assortiment des lignes qui compliquent les projections, en lui faisant saisir, du premier coup d'oeil, les diverses fonctions de ces lignes. […]» (Haüy, 1801, pp. lv–lvi)

In this short passage, Haüy consciously summarizes the scientific basis of projection in the eighty-six plates of his *Traité de minéralogie. He* explains the aim of line types (continuous or dash) for the edges of opaque polyhedra. As a demonstration of the skills gained in the field, Haüy draws a comparison with an observer's visual perception, evidently simplifying the reader's understanding. Other line types are used to highlight the geometric elements of the crystals. The fundamental geometries, the *molécules intégrantes* that determine the macro-shape of the crystal are fielded with various gradations of hatches, as well as the hemihedral shapes of the crystals that appear as opaque solids within the transparent major crystals. **(Fig. 7)**

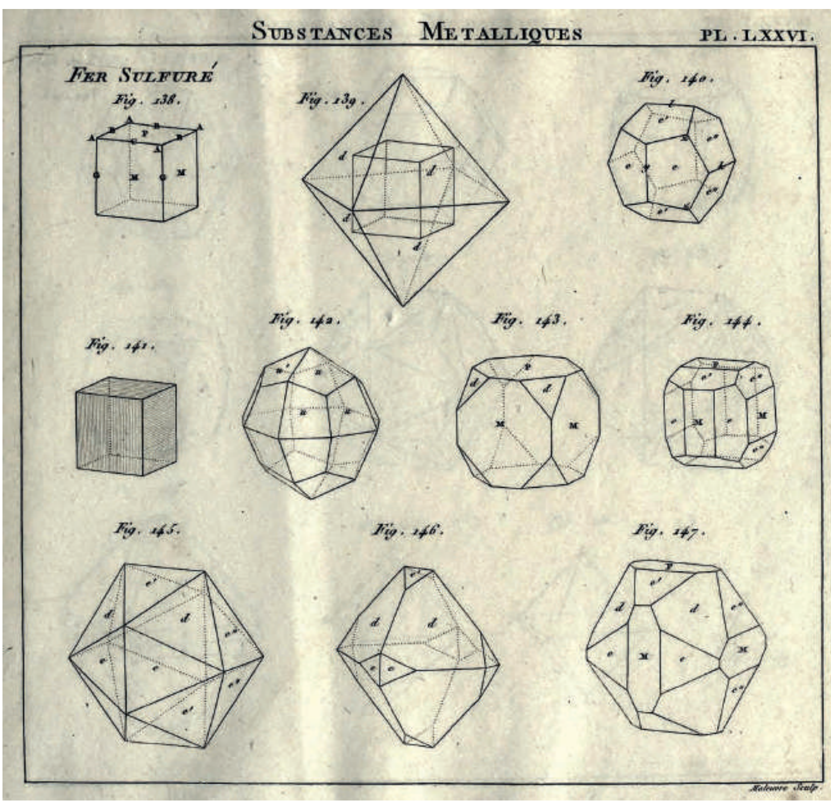


Fig. 7. René-Just Haüy, The crystal polyhedra of metals. (Haüy, 1801)

In his last Treatise (1822), Haüy was fully aware of the method of representation he had chosen – later known as Parallel Projections (Scolari, 2005) – so much so that he highlighted it in the title of the book, and he dedicated a specific chapter to it – *Méthode pour représenter en projection les formes des cristaux* (Haüy, 1822, pp. 583-647) – evidently to underline the definitive shift in crystal technical drawing. In this chapter, the author first clarifies the characteristics of the Improper Projection Center compared to Perspective one, a method widely known and codified then. Subsequently, he deepens the reasons for projection with an Infinite Center of Projection perpendicular to two distinct planes, horizontal and vertical.

«[…] Maintenant, si l'on imagine qu'um œil placé devant une perspective s'éloigne à l'infini, alors tous les rayons dont les empreintes formaient cette perspective, deviendront parallèles, et l'image produite sur le plan transparent prendra le nom de *projection*. On suppose que le mouvement en vertu duquel l'œil s'éloigne de ce plan, ait lieu dans la direction du rayon qui fait la fonction d'axe à l'égard de tous les autres, c'est-à-dire de celui qui passe par le centre du solide; et parce que l'on peut faire varier à l'infini la position de l'œil, et par une suite nécessaire la direction de l'axe dont il s'agit, il en résulte que l'on est le maître de supposer des directions quelconques aux rayons qui produisent l'image, pourvu qu'ils soient parallèles entre eux. C'est à ce genre d'empreintes que se rapporte la méthode que l'on a adoptée pour les constructions destinées à représenter les formes cristallines. […] parce qu'elle facilite l'étude de leurs formes.

Je ne dois pas omettre que comme nous ne considérons ici que des solides terminés par des faces planes, il suffit, au moins dans les cas ordinaires, de supposer que de tous les points de leurs diverses arêtes, il parte des rayons qui se dirigent vers l'œil. L'ensemble de tous ces points donnera la projection du solide dessinée au simple trait. De plus, parmi les diverses faces de ce même solide, les unes sont situées en avant, les autres par derrière. Ou marquera les premières par des lignes pleines et les autres par des lignes ponctuées.

Avant d'exposer les règles auxquelles est soumise, dans la pratique, la méthode de tracer les figures des cristaux, je vais donner une idée générale de la marche qu'elle suit pour arriver à son but. Je commence par mettre la forme primitive successivement en projection horizontale et en projection verticale. La première est celle qui est produite par des lignes abaissées perpendiculairement des extrémités de toutes les arêtes du solide sur un plan horizontal, avec la condition que les points où ces perpendiculaires rencontrent le plan dont il s'agit , soient joints deux à deux par des droites qui correspondent aux arêtes. Par exemple, la projection horizontale d'un cube dont deux faces opposées seraient parallèles au plan de projection est un carré. Dans le cas présent, tous les points des arêtes longitudinales se recouvrent mutuellement, en sorte que la projection de chacune se réduit à un simple point. Les deux bases se recouvrent de même, de manière que la projection de la base inférieure se confond avec celle de la base supérieure qui, seule, produit le carré auquel se réduit la projection totale du cube. La projection verticale est tracée d'après les mêmes conditions, avec cette différence, que le plan de projection est situé verticalement.

La projection horizontale sert comme de guide pour arriver à la projection verticale. […]» (Haüy, 1822, pp. 584-86)

Depending on the purpose of the representation, the vertical projection is split into projection nivelée and projection variée. To facilitate its application to crystals, Haüy calculates the numerical relations between the edges of the polyhedra and the sides of the figures projected onto the planes, as Farish (Farish, 1820) did two years earlier for isometric axonometry.

As already mentioned, it is not possible to exclude a bibliographic interweaving between Haüy and Farish – despite the critical relations between their countries – but certainly the French scholar placed more emphasis on the theoretical aspects of axonometric representation of the morphology of crystals, and less on the graphic aspects, proposing the construction of the drawings through numerical relationships among the edges of the polyhedra, among the dihedral angles, and also among the faces, similarly to the Farish procedure applied to solids of greater size and complexity.

Most of the *Méthode pour représenter en projection les formes des cristaux* chapter is also devoted to the graphic construction of double orthogonal projections, which is useful for studying the faces of polyhedra. This last aspect determines a cultural link between Haüy and Gaspard Monge, if not their direct contact. In this regard, it should be emphasized that Monge's lectures were well known in the polytechnic circles of Paris (Docci & Migliari, 1994), and that the book *Géométrie descriptive* was published two years before Haüy's first treatise.

«[…] Je reviens un instant à la comparaison de la projection avec la perspective, relativement aux formes cristallines. Dans la perspective, les rayons partis d'une face qui a ses côtés parallèles deux à deux, telle qu'un carré, un rhombe, un hexagone régulier ou symétrique, forment une pyramide dont la base coïncide avec la même face, et dont le sommet est dans l'œil du spectateur. Alors les lignes qui sont parallèles sur le cristal ne peuvent l'ètre dans la perspective que sous certaines conditions, comme lorsque les faces auxquelles appartiennent ces lignes sont parallèles au plan idéal. Dans la projection, les ligues qui sont parallèles sur le cristal conservent constamment leur parallélisme, à cause de la forme prismatique que prend l'ensemble des rayons partis d'une même face. La différence est sensible à la seule inspection des figures 1 et 2, pl. 74, dont la première offre la perspective d'un prisme hexaèdre régulier, et la deuxième sa projection. Parmi les six pans du prisme en perspective, ceux qui sont désignés l'un par ***gff'g'***, ***bcc'b'***, sont les seuls qui aient leurs côtés parallèles deux à deux. Mais le pan ***dff'd'***, par exemple, n'a que deux côtés parallèles, savoir: ***dd' et ff'***; les deux autres ***df', f'd'*** sont très sensiblement inclinés entre eux. La même observation s'applique aux pans ***cdd'c'***, ***abc'a'***, ***agg'a'***. Chacune des bases hexagonales, par exemple la base inférieure, n'a non plus que deux côtés parallèles, savoir: ***b'c'*** et ***g'f'***. Mais ***ab'***, ***d'f*** d'une part, et ***a'g'***, ***c'd'*** de l'autre, sont visiblement inclinés entre eux. Il n'en est pas de même des côtés qui correspondent aux précédens sur la projection du prisme (fig. 2), et qui tous remplissent la condition du parallélisme. Il en résulte cet avantage, que l'aspect géométrique de la projection est beaucoup plus conforme que celui de la perspective à l'idée que conçoit le cristallographe du solide qui est le sujet de l'une et de l'autre, et au résultat des mesures mécaniques prises immédiatement sur ce solide. […] (Haüy, 1822, pp.588-89)

Pour en venir maintenant aux applications de la méthode, concevons que le rectngle ***orr'o'*** (fig. 6) représente le plan de projection horizontale , et que or soit la section de ce plan avec le plan vertical. Ce dernier est indiqué ici par le rectangle ***orr"o"***, qui est de niveau avec le plan horizontal; en sorte que pour se le représenter dans sa vraie position, on doit concevoir qu'il se, relève en tournant sur la ligne or, jusqu'à ce qu'il soit perpendiculaire au plan ***orr'o'***. Mais nous verrons que la coïncidence des deux plans en un seul ne change rien, an résultat qui aurait lien si les deux plans étaient à angle droit l'un sur l'autre.

Le corps que l'un se propose de représenter est supposé être situé à une certaine hauteur, au-dessus du plan de projection horizontale, vis-à-vis celui de projection verticale. On imagine un troisième plan qui est transparent, dont là ligne ***o'r'*** est la section avec le plan horizontal sur lequel il est perpendiculaire. Je donne à ce plan le nom de ***plan idéal***. On suppose que des rayons parallèles partis des différens points du solide passent à travers ce plan, en y laissant chacun leur empreinte, et le but de l'opération est de tracer une copie fidèle de l'image produite par la somme de ces empreintes. Mais cette copie étant un dessin au simple trait, n'est composée, comme je l'ai déjà dit, que des lignes qui terminent les différentes faces du solide, et dont les unes qui appartiennent aux fiices antérieures doivent être pleines, et les autres qui se rapportent aux faces de la partie opposée doivent être ponctuées. Je rappellerai ici ce que j'ai dit en comparant l'effet de la projection avec celui de la perspective; savoir, que dans l'une et l'autre l'image produit sur l'oeil une impression semblable à celle qui naîtrait de la vue immédiate de l'objet, avec cette différence que dans le cas de la perspective l'œil est placé à une distance finie du plan idéal, au lieu que dans le cas de là projection, il est censé en être éloigné à une distance infinie. […]

Supposons ce rhomboïde situé de manière que son axe soit dirigé verticalement, et que l'une de ses coupes principales coïncide avec un plan perpendiculaire au plan idéal. Il est aisé de voir que sa projection horizontale sera l'hexagone 36 (fig. 6), dans lequel les trois rhombes 5671, 723i, 5431 seront les projections de ceux qui appartiennent au sommet supérieur, et les trois rhombes 6728, 2348, 4568, celles des rhombes contigus au sommet inférieur; et la droite 613, dont le prolongement est perpendiculaire sur ***o'r'***, sera la projection de la coupe principale désignée ci-dessus. […]» (Haüy, 1822, pp. 594–600) **(Fig. 10)**

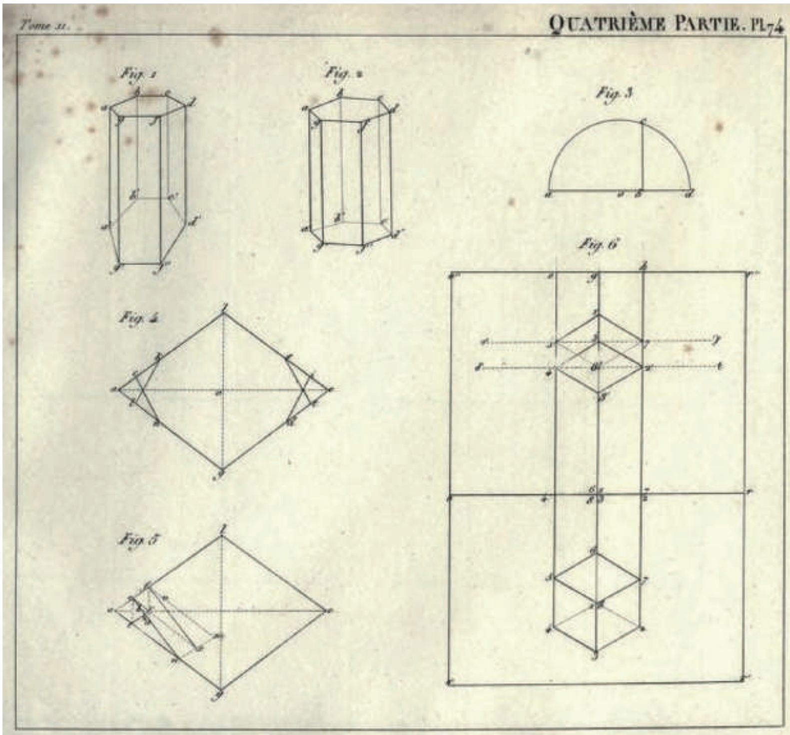


Fig. 10. René-Just Haüy, Comparison between the perspective and the axonometry of a crystal in the shape of a prism with a hexagonal base (figs. 1, 2); A rhomboid in double orthogonal projection (Haüy, 1822, plate 74)

As already described in the first chapter of this book, Haüy's studies are also based on the intuition of the geometric matrix of crystals in direct relation to the theories of Niels Stensen (1638-1686): the *molécules intégrantes* that determine the shape of the crystal by geometric juxtaposition of small "molecules" corresponding to its chemical composition. Also in this case, the author elaborated a specific graphic representation, called "blocks", to give a graphic and visual character to his scientific intuition. This theory was soon superseded by the concept of the "crystal lattice" by Auguste Bravais (1811-1863) which was represented mainly in Parallel Projections in his *Études cristallographiques* (Paris 1851). **(Ch1. Fig. 7)**

Attesting that Rome de L'Isle's and Haüy's research led to a paradigm shift in Minerals, the tables attached to their Treatises can be considered nodal references in the iconographic analysis of crystal representations.

On the other hand, an interesting comparison can be made between the drawings developed by the two French scientists in about fifty years and the iconographies of the *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers* by Denis Diderot, Jean-Baptiste le Rond d'Alembert (1751-1772)[6]. This is especially true since the entire

6 The French encyclopedia has been consulted thanks to (*Édition Numérique Collaborative et CRitique de l'Encyclopédie Ou Dictionnaire Raisonné Des Sciences, Des Arts et Des Métiers de Diderot, D'Alembert et Jaucourt (1751-1772)*, 2017).

encyclopedic work precedes the five fundamental books of Rome de L'Isle and Haüy. Therefore, the two scientists most likely studied the entries and tables, similar to their research.

As is known, the French encyclopedia is divided into twenty-eight volumes, of which seventeen are dedicated to entries, and eleven devoted to illustrations and detailed captions (Pietrabissa, 2023).

The entries and related illustrations most akin to the studies of Rome de L'Isle and Haüy may be: *Minéralogie* (vol. X, 1765, pp. 541-43), and *Crystal* (vol. IV, 1754, pp. 523-26) given the lack of the word "crystallography", probably introduced by Rome de L'Isle with his first book (1772).

Within this text's limits, the *Encyclopédie*'s consultation can be enriched with other entries linked to the first two through etymological roots.

Mineralogy is defined as a branch of the *Histoire Naturelle* – linked to taxonomy by Carl von Linné – focused on the knowledge of the substances of the mineral kingdom: earth, stones, salts, flammable substances, petrifications, «en un mot, des corps inanimés & non pourvus d'organes sensibles qui se trouvent dans le sein de la terre & à sa surface». A detailed dissertation with links to various mineral-specific entries follows this.

The complex subject is illustrated in 47 plates (vol. VI, 1768), drawn by the little-known artist de la Rue, and divided into six sections: *Corps étrangers au Regne minéral, qui se trouvent dans la terre; Pierres crystallisées; Mines crystallisées; Montagnes; Glaciers; Volcans, Solfatare & Pavé des géans; Filons, Mines & travaux des Mines*. As we leaf through these pages today, we see that an exhaustive range of examples on the subject and the methods and techniques of representation chosen for Mineralogy is possible. In summary, the figurative drawing in chiaroscuro is predominant. Pseudo-axonometric drawings are used for some specific cases: the fossil remains of plants or small vertebrates are drawn in dimetric pseudo-axonometry starting from the orthogonal projection of the surface of the sectioned stone (a necessary cut to show the find incorporated into the rock), from which vertices extend as many parallel segments (or almost) to give depth to the representation (*Corps étrangers au Regne minéral* […], vol. VI, 1768, plates V, IX-XIV).

The graphic assonances among the iconographies of the *Encyclopédie*, those by Rome de L'Isle and those by Haüy, are evident in the nine plates on the *Pierres crystallisées*. The draftsman gives an inductive visual approach to representation: the crystallographic groups are drawn from life with accurate variations of chiaroscuro, and the isolated crystals are in pseudo-axonometry with hatching gradations to highlight the faces' inclinations. **(Fig.11)**

Fig. 11. Figurative drawings of groups of crystals, and isolated crystals in pseudo- axonometry. (Diderot & d'Alembert, 1768, plate I)

In particular, in plate VI (*Histoire Naturelle. Crystallisations*), de la Rue composed thirteen drawings with the aforementioned inductive approach. Among these, the *Crystal d'Islande* has more accurate axonometric characteristics than all the others of the same entry **(Fig. 12)**. The parallelepiped image has all the edges parallel; the three visible faces are hatched with parallel lines with three density variations; the edges of the "hidden" faces are outlined with dash lines, just as Haüy did in 1801. The major edges are quoted with values 6 and 8, as reported in the article of the *Crystal d'Islande* entry «[…] c'est un parallélipipede composé de 6 parallélogrammes & de 8 angles solides, dont 4 sont aigus & 4 obtus […]» ('Crystal d'Islande', 1754).

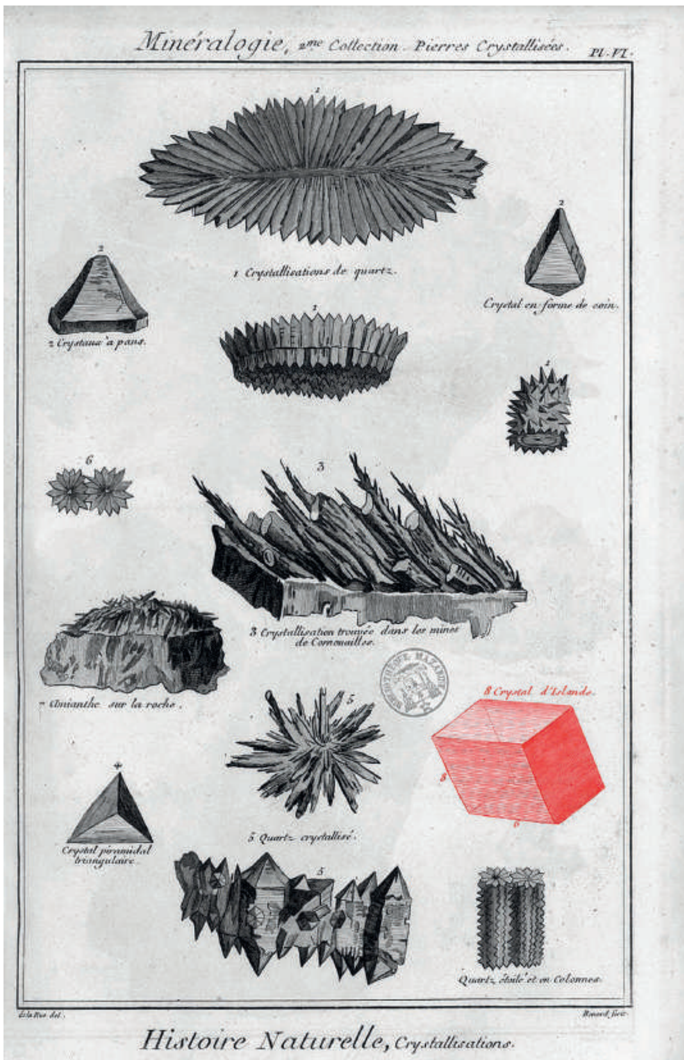


Fig. 12. Crystal d'Islande (Diderot & d'Alembert, 1768), drawn in pseudo-axonometry (red texture), among other figurative drawings; digital elaboration by P. Argenziano, A. Avella, and N. Pisacane, 2024.

This exception to the usual graphic design of *Minéralogie*'s plates suggests that the draftsman was more aware of the objectifying character of that graphic method than others about the unequivocal interpretation of dimensions and proportions. Unsurprisingly, there are no examples of dimensioned perspective drawings [7]. Since the edges of this crystal do not respect the geometric proportions indicated in the entry, it is evident that the conscious application of axonometry to technical representation must wait for Farish's study, but certainly the graphic message at least in the field of crystallography has been defined.

It is necessary to wait for the plates of *Traité des caractères physiques des pierres précieuses* (Haüy, 1817) and of *Traité de cristallographie* (Haüy, 1822) to appreciate the graphic and the theoretical awareness of the Parallel Projections method, applied to the technical representation of gems and crystals, as discussed above.

In the representation of minerals and crystals, the Orthogonal Projections method is rare, as we have seen, in the 47 plates on Mineralogy; However, it is widely used in many other entries of the French encyclopedia. Therefore, it is evident that this method was widely known, well before the publication by Gaspard Monge, and that it was used to attribute tabular rigor to technical representations.

In the scientific field of crystal representation, two of the eleven plates of the Orfèvre-jouaillier, metteur en oeuvre entry 20, are worth mentioning; in the plates, *Brillans Rares* (*planche* I) and *Taille des Diamans* (*planche* II), an interesting variety of gems, existing and designed[8], are drawn in double and triple orthogonal projections.

The attentive reader knows the profound relationship between a crystal's chemical and atomic structure and the geometrical variables of its transformation design into a gem, as intuited by Haüy as early as 1817 **(Figg. 8, 9)**. For these reasons, the comparative reading of the drawings of crystals and faceted gems is based on the same principle.

7 The graphic analysis of the proportions among the edges of the parallelepiped shows that the draughtsman attributed the same distance to the dimensioned ones, less to the third direction; The ratio is about 6:5,6:4.

8 The captions of the plate I are very detailed, right down to the indication of the carat weight of the four diamonds. On the contrary, those of plate II are limited to enumerating the number of facets of the twenty diamonds. The indication of the weight of the diamonds only in the first case, would still confirm that the second are only designed diamonds.

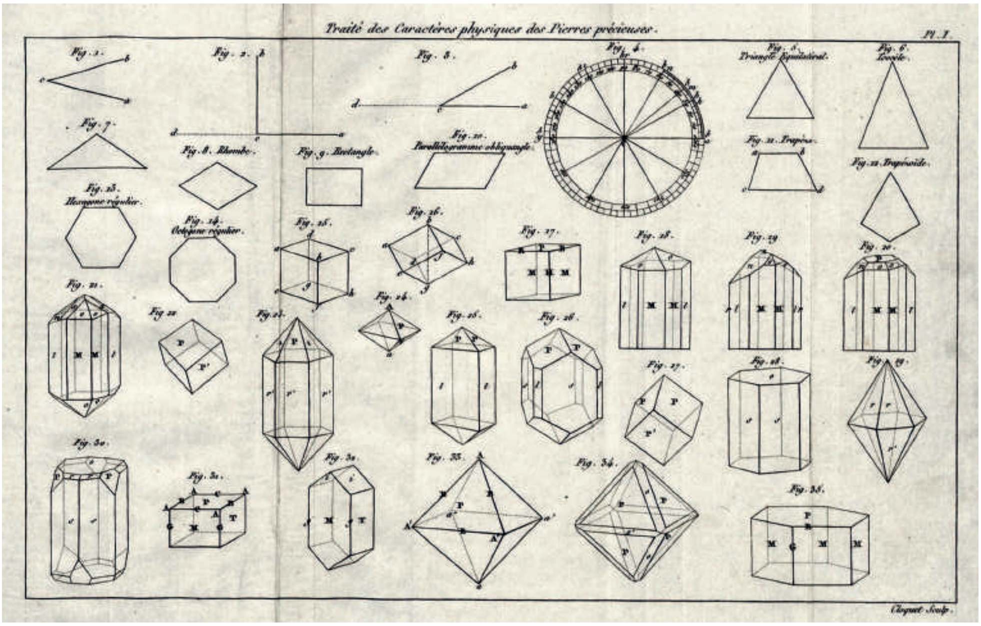


Fig. 8. René-Just Haüy, Classification of gemstones, as crystal polyhedra. Representation of faces, and Axonometry with parametric dimensions. (Haüy, 1817, plate I)

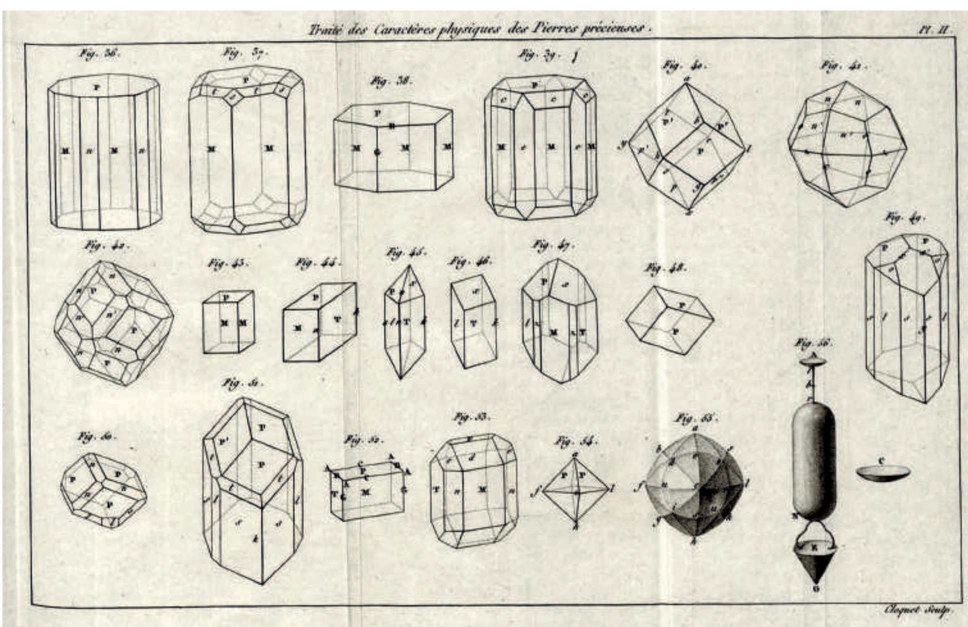


Fig. 9. René-Just Haüy, Classification of gemstones, as crystal polyhedra. Axonometry representation with parametric dimensions, as opaque solids and chiaroscuro ones. (Haüy, 1817, plate II)

In the plate I of the *Orfèvre-jouaillier, metteur en oeuvre* entry, there are the four most precious diamonds, among those known in the mid-Eighteenth Century: the Grand Mughal, the Florentine Diamond, the Grand Sancy, and the Regent Diamond. Since those diamonds were in the private collections of various European noble families, it is presumable that the draughtsman Jacques-Raymond Lucotte (1733-1804) drew up the drawings based on earlier iconographic sources – such as, for example, the writings of Jean-Baptiste Tavernier (1605-1689) – and not on direct contact with the gems.

In plante II, twenty diamonds are drawn, in double and triple orthogonal projections, according to a bustrofedic distribution, starting from the top right vertex of the table. Following the distribution of the drawings in the plate, the number of facets of the gems increases – from 10 to 117 – and the morphological and compositional characteristics of the gems vary: the first ten have no table; The polygon girdle is in numerical progression, and differs from regular to irregular sides; The facet of crowns is becoming more and more complex. In this plate, the draughtsman tests several faceting, following the geometric progression rather than by the gem feasibility, as has been demonstrated in a previous study using digital three-dimensional modeling based on the dimensions extracted from the planche **(Figg. 13, 14)** (Pisacane et al., 2023b).

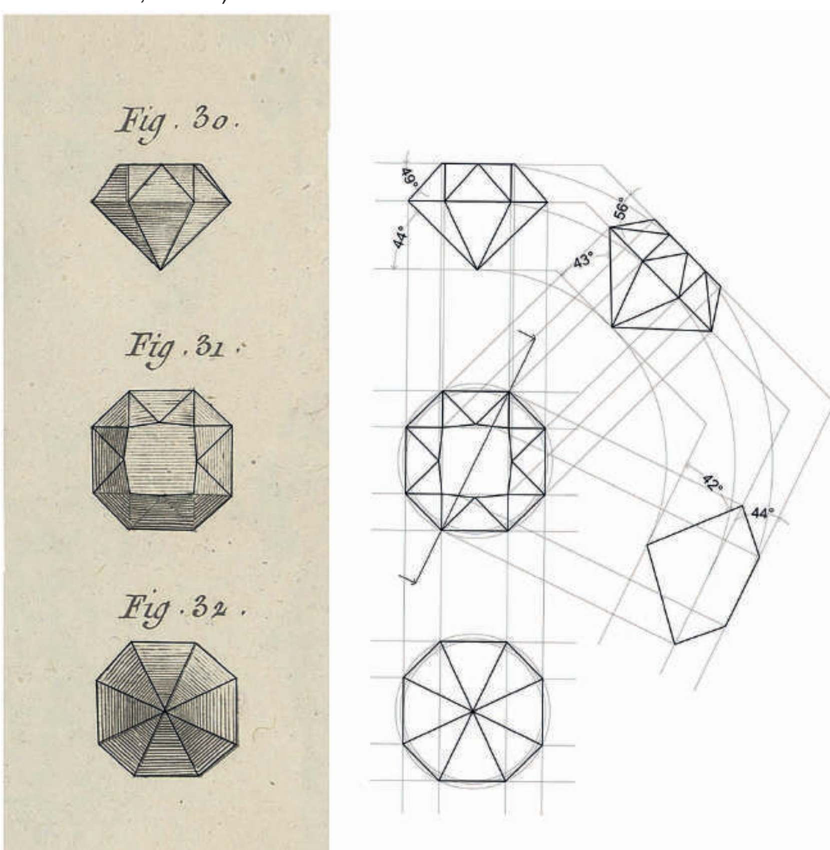


Fig. 13. Graphic-geometric analysis of drawings 30, 31 and 32 in the Plate II Orfèvre Jouaillier, Metteur en Oeuvre, Taille des Diamans. Drawing by P. Argenziano, A. Avella, and N. Pisacane, 2023.

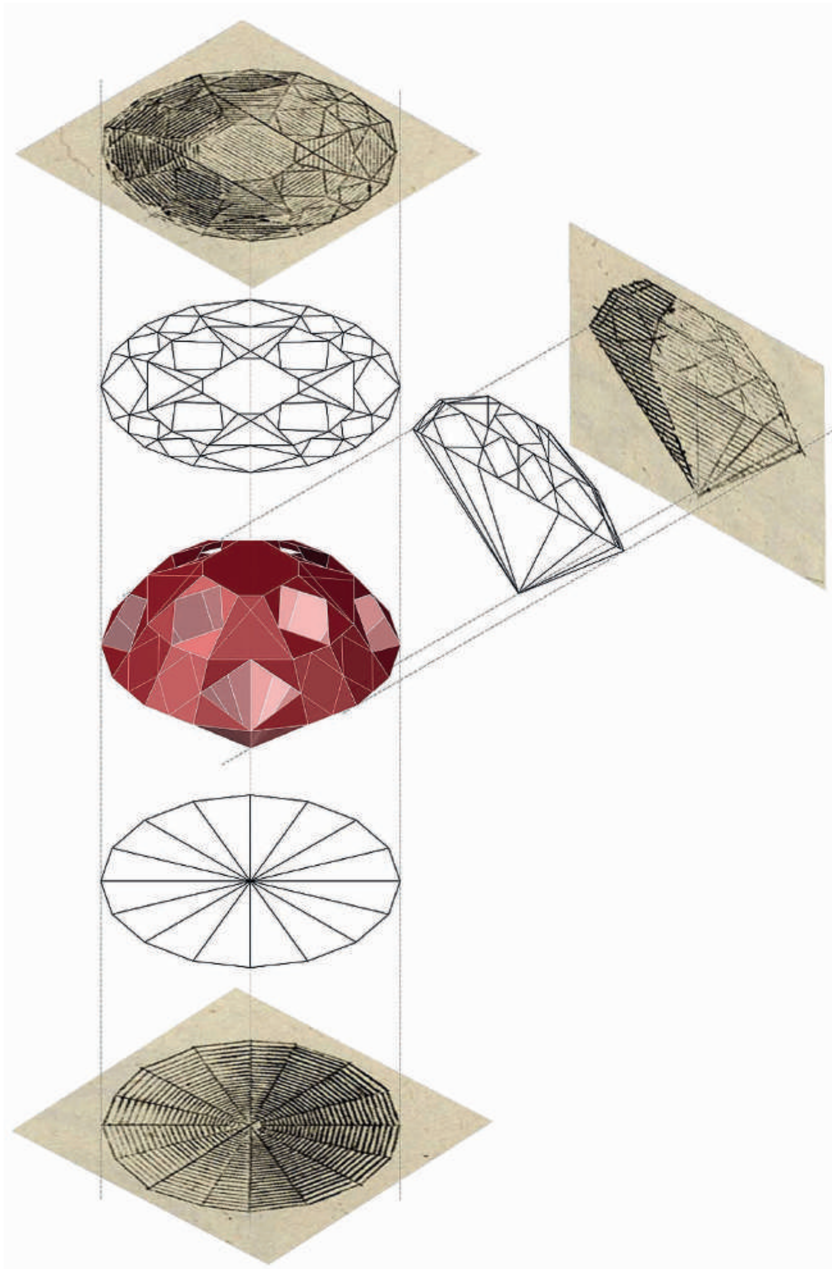


Fig. 14. Surface geometric modeling of cut 48, 49 and 50 in the Planche II Orfèvre Jouaillier, Metteur en Oeuvre, Taille des Diamans. Drawing by by P. Argenziano, A. Avella, and N. Pisacane, 2023.

The comparison between these plates highlights first of all a rigorous approach to the drawings: the twenty-four faceted gems are drawn in orthogonal projections and the respective views are organized in a tabular manner according to the vertical axis of symmetry; The horizontal distribution follows the apparent ground line of the vertical projections and the axis of symmetry of the horizontal projections.

This rigorous graphic approach is reflected both in the plates' captions and in the position of the architect J.R. Lucotte – trained in the *Académie royale d'architecture* – draftsman of the plates of the three "Orfèvre" entries, and collaborator of the *Encyclopedie* in several entries and images. The first view of the gems is always an "*elévation*", so below in vertical alignment is placed the view called "*plan de dessus*", and for the more complex gems, even lower is placed the third view, called "*plan du dessous*". In fact, Lucotte applied the graphic layout of architectural drawing to gems, according to the practice taught of the French academy, and found in the pages of the *Encyclopedie*[9]: the elevation and plans of a building, as of any object, are always outlined in reciprocal relationship.

This practice will find its geometric concreteness in Gaspard Monge's text, and a specific application to crystals (and therefore to faceted gems, taking up Steno) in Haüy's latest book.

STEREOGRAPHY AS A METHOD OF CRYSTAL REPRESENTATION

The echo of Haüy's early studies crossed the borders of France; in the years, when he was studing for his last publication, various mathematicians in Germany approached the subject of geometric crystallographer from a different point of view.

Christian Samuel Weiss (1780-1856) introduced a method of morphological classification based on the position of the faces, and on the direction of the axes of symmetry of crystallographic polyhedra (Weiss, 1809) that would be the inspiration for the stereographic projection applied for the first time to crystals by Franz Ernst Neumann (1798-1895) (Neumann, 1826). Simultaneously to Haüy, another method of representation of crystal polyhedra, certainly more adherent to the possible morphological variables, because it is independent of the crystallographic system, compared to the axonometric representation proposed by Haüy.

Neumann's crystallographic stereography considers the projection surface, the plane of one face of the crystal polyhedron, or the surface of the sphere whose center is at the origin of the perpendiculars to the faces. Thus, the faces of the crystal are represented by the points (poles of the faces) of the intersection of their perpendiculars with the projection surface.

9 The graphic layout of the façade and of the plans in architectural drawing, still detached from the theoretical studies by Gaspard Monge, can be found in the entries *Architecture* (vol. I, 1751, p. 617-18), *Dessinateur, en Architecture* (vol. IV, 1754, 894), *Elévation* (vol. V, 1755, p.505), *Ichnographie* (vol. VIII, 1765, p. 481), *Orthographie* (vol. XI, 1765, p. 670), *Plan* (vol. XII, 1765, p. 692).

William Hallowes Miller (1801-1880) developed and simplified this projection method in his *Treatise on Crystallography* (Cambridge-London, 1839).

«[…] The use of this method [e.d. Neumann one] led to the substitution of spherical trigonometry for solid and analytical geometry processes in deducing expressions for determining the positions of the feces of crystals and the angles they make with each other. The expressions which in this Treatise have thus been obtained, are remarkable for their symmetry and simplicity, and are all adapted to logarithmic computation. They are, it is believed, for the most part new. For the convenience of calculation the position of one face concerning another is represented by the angle between perpendicular to the faces, or by the supplement of the angle between the faces, according to the commonly received definition of the angle between two of the planes that bound a solid. […]» (Miller, 1839, pp.3-4)

Considering the origin of the crystallographic axes as the center of an ideal sphere, the symmetry elements of the polyhedra are projected from the two extremes of the vertical axis (poles) onto the equatorial plane (Miller, 1839, pp.1-26). A symbolic encoding identifies the images about their projection center, and to the degree of symmetry: X and O identify the projections from the zenith and nadir, respectively; An 'almond', a triangle, a square, a hexagon respectively identify the elements of binary, ternary, quaternary and senary symmetry. Depending on the position of the planes of symmetry about the two poles, the traces can be diameters, or arcs subtended by the diameters of the maximum circumference. The faces of the polyhedron are consequently projected onto the equatorial plane and are identified according to a unique numerical coding, introduced by Miller himself (Millerian indices). **(Fig. 15)** (Miller, 1839, pp.129-139)

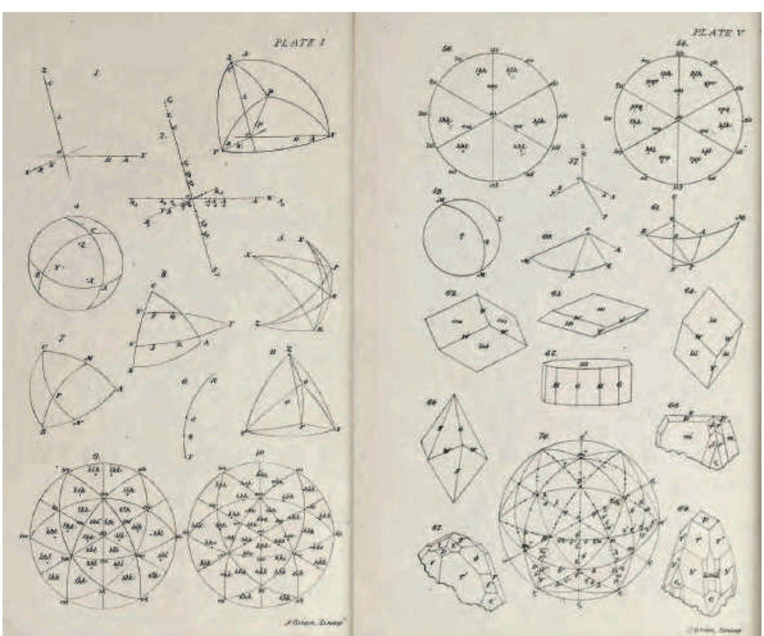


Fig. 15. Crystal Stereography Applications. (Miller, 1839, plates I, V)

Drawing from life, Axonometry, Orthogonal Projection, and Perspective were first used empirically, then scientifically in about three centuries of research in the Earth Sciences.

Scientists have investigated the solids' forms –amorphous, irregular, or regular – by retracing in their own way the steps Euclid took in the Elements (books XI, XII, XIII) (Acerbi, 2019, pp. 389–399). Then, as in recent times, there is a need to know the peculiar characteristics of polyhedra through the dimensions of the faces, of the dihedral angles between them, through the operations of direct measurement and projection on the plane.

In other words, to study the forms in the three-dimensional space through Drawing, the only medium of analysis that realizes critical thinking through the eye that observes and controls the space, the plane of representation, and the hand that draws.

CONCLUSIONS

In addition to the main topic of the book, our research on Gemstones' Drawing/ Design opens a new scenario: Jewel Information Modelling (JIM)

The technological solution we intend to develop is applied to Made in Italy jewelry, existing and projected, according to the digital paradigm of the blockchain. JIM's innovative character is the systematization of the logical digital protocols of the blockchain and three-dimensional computer systems for data management (information modeling), with applications in the design and analysis of Italian handcrafted jewelry.

Concerning the logical structures of Information Modelling and blockchain, the JIM consists of a genesis block (3D model) to which as many main blocks are connected as there are quantitative and qualitative data that characterize the metals and gems that constitute it.

In the case of the existing jewel (**Fig. 1**), the genesis block is made using 3D technologies for scanning and photorealistic digitization of design objects – according to the scientific protocols of the discipline of Design – and the connected main blocks contain the data of the scientific characterization of metals and gems (mineral or organic) – acquired through non-invasive digital technologies (Argenziano et al., 2022). Other main blocks contain the qualitative data of the historical-critical and aesthetic-functional analysis of the jewel and those concerning ownership, care, maintenance, and conservation.

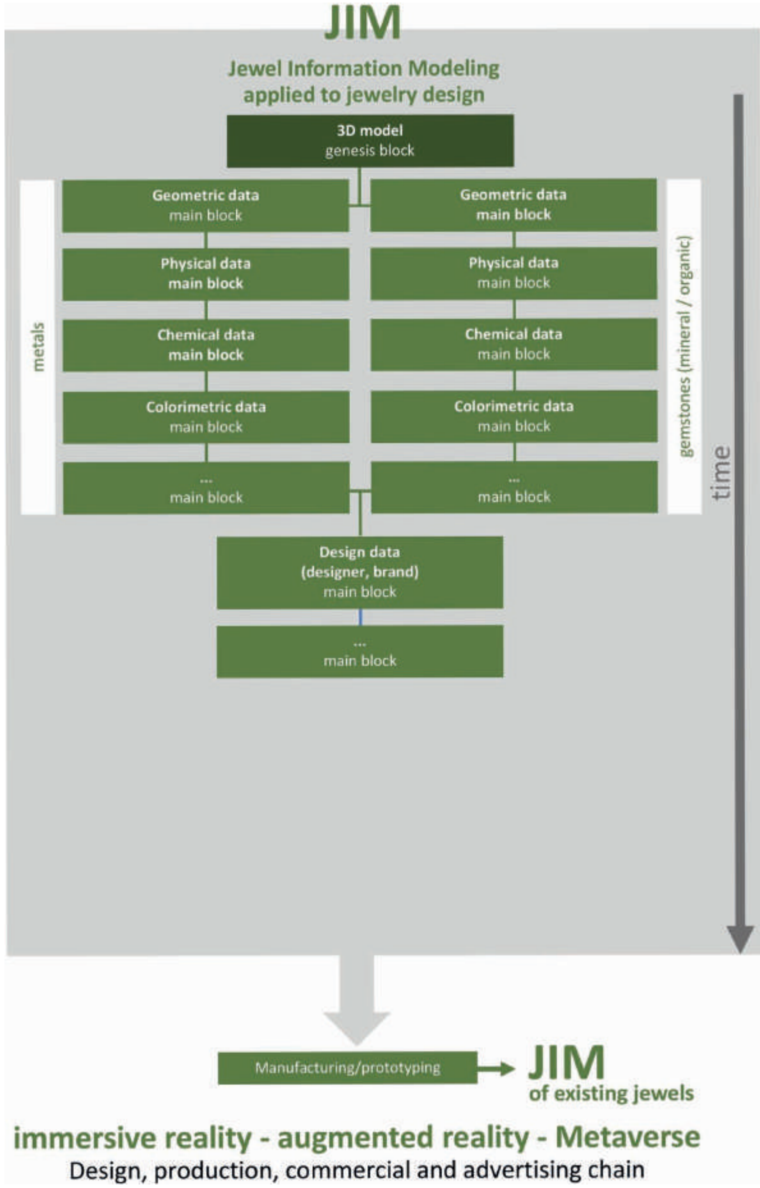


Fig. 1. Flow-chart of designed Jewel Information Modelling, by P. Argenziano, A. Avella, and N. Pisacane, 2024.

Thus, the existing jewel is cloned into a digital twin whose associated data, encrypted in blockchain, can be consulted through immersive and/or augmented reality technologies and devices, such as the Metaverse. This can always be implemented according to the interoperability approach of Information Modelling. The resolution of the 3D model and the number of data classified in the JIM can be selectively recorded about the characteristics of the jewel and/or in proportion to its commercial, historical-artistic, and sentimental value.

In the design field (**Fig. 2**), the genesis block of the JIM is the photorealistic 3D virtual model of the jewel, and the connected main blocks contain the quantitative and qualitative data of the project and of the metals and gems that compose it.

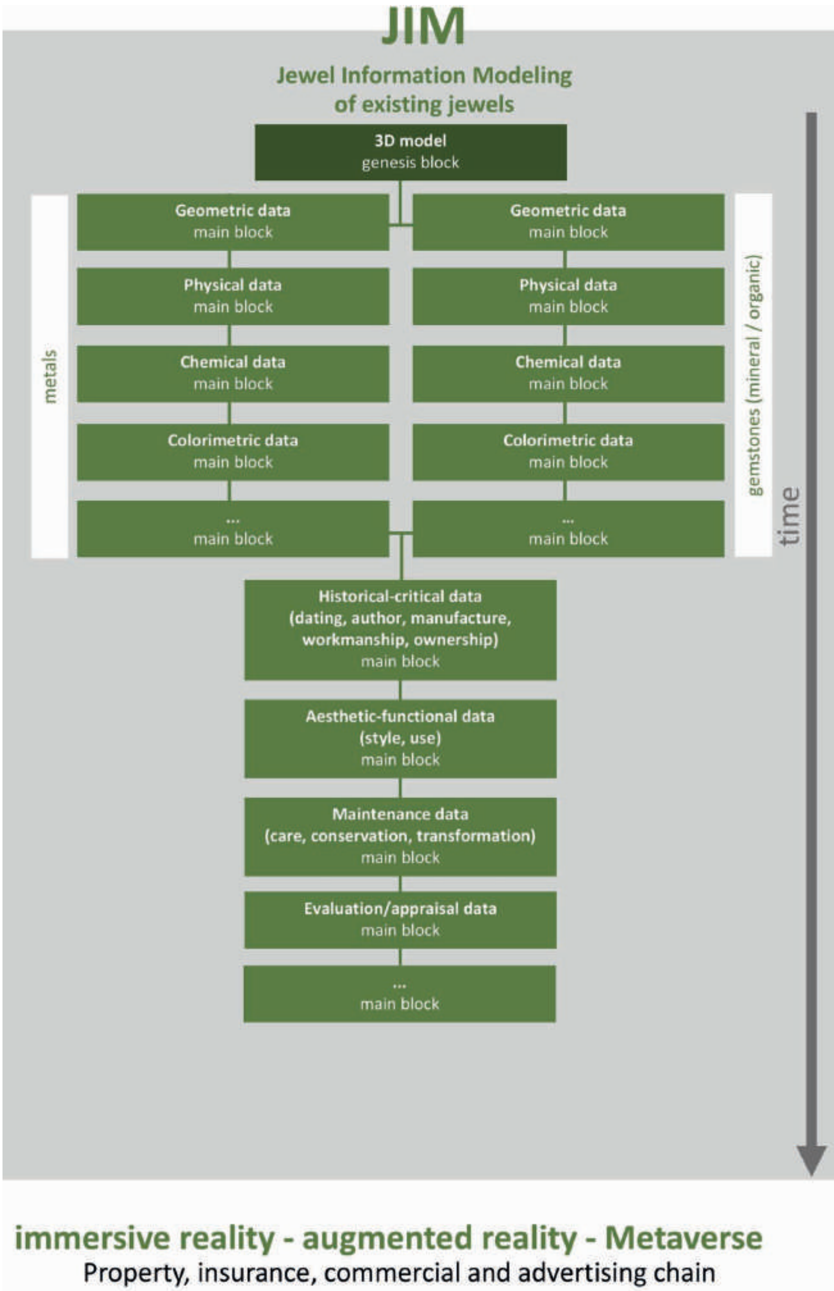


Fig. 2. Flow-chart of existing Jewel Information Modelling, by P. Argenziano, A. Avella, and N. Pisacane, 2024.

The designed jewel exists in the virtuality of the JIM. It can be worn and admired through immersive and augmented reality technologies and devices, such as the Metaverse, thanks to which it is possible to consult the quantitative and qualitative data of the project also in terms of product customization. The JIM of the jewel designed and customized can be made on request in the Italian production chain as a unicum to which the relative blockchain is connected.

The JIM could be a technological solution to enhance the reference market of designers, artisanal realities, and small and medium-sized jewelry companies in Italy, thus addressing experts in the sector, collectors, and individual users with different levels of use and interaction. (Prim, 2021)

The main purposes of the existing and planned JIM are to appreciate and certify jewelry and its components on a technical-scientific basis, with application repercussions in the patrimonial, insurance, design, production, commercial, and advertising supply chain.

The JIM protocol can be extended to international realities at the end of the experimental phase.

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