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ABOUT THE PROPOSAL TO CALCULATE THE LOCAL SOLAR DAY

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Abstract: In a previous publication (Carrillo, 2021), some concepts necessary for the use of the local solar day concept were defined. In this article we will advance in numerically obtaining the time of day in which the local solar day concept provides radiation, taking into consideration, the concepts previously constructed such as hour-day index, theta ith hour, maximum theta, initial minimum theta and local minimum theta and we will connect them with a function that adjusts the radiation for a day in which the radiation is given with a zenith up to 90°. It will be an essential requirement to know the angle of the sun's radiation at each intermediate instant, in which the observer wants to know the time of said day, so a transducer is proposed that measures that angle. In addition, the 4th order and 6th order models are compared using the quadratic difference. And then, it is integrating the mentioned difference to know the total difference during the day.

INTRODUCTION

The local solar day has several important parameters in the day, we will consider the concepts:

θ_{mini}, θ_{max}, θ_{minif} where these values satisfy the following conditions:

$$0 \leq \theta_{mini}, \theta_{minf} \leq \theta_{max} \leq \frac{\pi}{2}$$ in Equation 1

The meanings of these variables are as follows and a graphic approximation of them is given in Figure 1.

θ_{mini}: Minimum initial theta, which is the angle measured with respect to the horizontal that is between a projected point placed on a vertical line and its shadow that falls on the horizontal plane.

θ_{minif} : Finishing minimum theta, is the smallest angle up to which the shadow of the projected point placed on a vertical line is formed and its shadow falls on the plane after the afternoon.

θ_{max}: Maximum Theta is the greatest angle for which during a certain day the shadow of the point is projected, placing said points on a vertical line and their shadow is that of the zenith of that day. This being less than or equal to 90°.

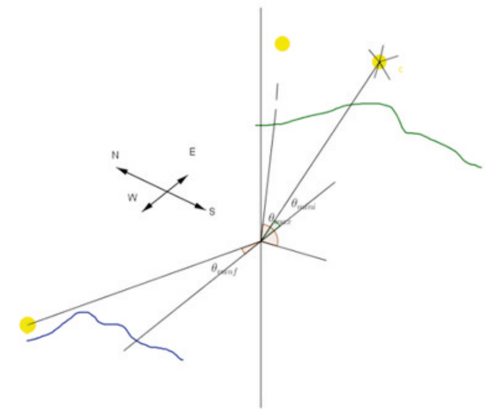

Figure 1: The approximate meaning of the parameters: θ_{mini}, θ_{minif}, θ_{max}

Now the parameter defined in [1] also states the fact described in inequation 2

$$0 \leq horai \leq 6$$ Inequation 2

And that the detected angle satisfy

$$\theta_{mini} o \ \theta_{minf} \leq \theta horai \leq \theta_{max}$$ Inequation 3

And the direct relationship between:

$$\theta horai = \theta_{max} * \frac{horai}{6}$$ Equation 1

It allows you to define the values of with the equations 2, 3

$$\theta_{mini} = \theta_{max} * horai_{mini}/6$$ Equation 2

$$\theta_{minf} = \theta_{max} * horai_{minf}/6$$ Equation 3

With the concepts of $horai_{mini}$, $horai_{minf}$, we ensure the fact that the zero hour will be considered the one in which there is radiation completely parallel to the plane where the measurement of the solar day is taken into account, even if at that moment there is no radiation incident through the local-geophysics conditions, thus, in fact a day according to that conception can begin to pass with a delay with respect to hour zero and similarly end at a time different from hour 12 if we consider it as measured with respect to the concept, *ihoradia*, for which its values range from 0 to 12.

Through these new definitions, new relationships can be established that involve, on the one hand, the values of θ and, on the other hand, those of horai as well as the derivatives of both.

$$\theta_{mini} \leq \theta horai \leq \theta_{max} \text{ or } \theta_{minf} \leq \theta horai \leq \theta_{max}$$

Inequation 4

$$horai_{mini} \leq horai \leq 6 \text{ or } horai_{minf} \leq horai \leq 6$$

Inequation 5

For convenience we can name the values of *θ horai* and to those of *horai* with a sign plus if its values are in clear growth or the sign minus if the values decline

Also, the relationship between. θ_{mini}, $\theta_{max,}$ $horai_{mini}$, θ_{minif}, $horai_{minf}$ allows us to obtain given the measure of θ_{mini}, θ_{minif} define what time the sun appears and what time it sets: $horai_{minf}$.

Let's assume a continuous sensor that systematically gives us the θ values and an integrated differentiator that allows us to locate when: *θ* grows or decreases. Thanks to this sensor we can determine with some precision the value of θ_{max}. (Although θ_{max} must not be known beforehand at the beginning, but an approximate value is given to the previous day or 90° in any case.)

From the same Equation 1 we can determine the local time: *horai*, If we use the equivalence between this from the Equivalence of radians to degrees, we obtain Equation 4

$$horai = 6\frac{\theta}{\theta_{max}} = 6\frac{rad}{rad_{max}}$$

Equation 4

Under this assumption we have that the value of maximum horai is 6 and coincides when the maximum angle is taken as a measure, and also the minimum values of the sunny hour are related to the $horai_{mini}$, $horai_{minf}$ so really the duration of the sunny day as stated in that Equation is given as in Equation 5

$$hours\ \ day = 6 + horai_{mini} + 6 - horai_{minf}$$
$$= 12 - (horai_{mini} + horai_{minf}).$$

Equation 5

and the precise time of day is obtained through the Equation applicable to the moments of solar radiation, which can be determined experimentally, for a given day, with a certain precision.

$$ihoradia = \chi_{horai+}\, horai + \chi_{horai-}(12 - horai)$$

Equation 6

Where:

$$\chi_{horai+} = \{1\ if\ horai\ grows;\ 0\ otherwise\}$$

Equation 7

$$\chi_{horai-} = \{1\ if\ horai\ decreases, 0\ otherwise\}$$

Equation 8

Solving for Equation 1 horai we have

$$horai = \frac{\theta horai}{\theta max} * 6$$

Equation 9

And substituting Equation 9 in 6 we have Equation 10 where the concept of hora-zenith appears

$$horazenithtali = \chi_{horai+} \frac{\theta horai}{\theta max} * 6 + \chi_{horai-}(12 - \frac{\theta horai}{\theta max} * 6)$$

Equation 10

Clearly the hora-zenith concept is a function of *horacenital*(*θhorai, θmax*) and takes values within the interval (0,12) with hour 6 coinciding with the zenith in *θmax*. This concept is directly linked to the appearance of solar irradiation directed towards a detector of the inclination angle of solar radiation. And with the irradiation data it allows us to know $horacenital_{in}$, $horacenital_{fin}$ which are obtained by substituting in: *θhorai* a $\theta horai_{mini}$, $\theta horai_{minf}$

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The formulas used to calculate the values of these concepts appear in table 1.

Concept derived from assuming that the zenith is on a specific day in θmax of that day and assigning the *horai* from 6 which is noon.

EXPERIMENT:

The measurements made in the city of Tehuacán are taken as data in a place where solar irradiation does not begin practically at zero since this is very difficult due to the orography of the place. In this case Tehuacán is in the Tehuacán valley, and the mountains on the eastern side prevent direct solar irradiation at small angles. Measurements carried out there are reported by [2]

But let's return to the problem of determining irradiation, which has been adjusted by the curve fitted to Holman's data by two models that intend for said phenomenon to occur symmetrically with respect to the instant in which the zenith is reached on that day.

Jared found the models of order four and order 6 that adjust Holman's solar irradiation considering the angle between 0° and 90° and reflected with respect to ninety increasing the values up to 180° although the experimental values are between 15° and 165°, according to this criterion.

Four-order model for Solar irradiation [1]

$$I = 2.97756 \times 10^{-7}t^4 - 0.000107307t^3 + 3.72174 \times 10^{-11}t^2 + 1.740032243t - 7.96336983$$ Equation 11

Order Model Six for Jared Solar Irradiation 2020

$$I = 2.9349 \times 10^{-11}t^6 - 1.5846 \times 10^{-8}t^5 + 3.7963 \times 10^{-6}t^4 - 0.000511t^3$$ Equation 12

In that description of irradiance, the variable t is measured in degrees, for both equations.

If we rename them as I^4 and I^6 we calculate the discrepancy between the two models for the data that belong to the domain of said functions, according to the source data.

This is done in the following way:

$$Error_q = \int_{\theta_i}^{\theta_f} \left(I^{(4)} - I^{(6)}\right)^2 dt = \int_{\theta_i}^{\theta_f} \left(\sum_{i=0}^{6} (a_i - b_i)t^i\right)^2$$

$dt \text{ with } a_5 = a_6 = 0$ Equation 13

If we define: $c_i = (a_i - b_i)$ we can calculate the square of the polynomial [1]

$$p(t) = \sum_{i=0}^{6} c_i t^i$$ Equation 14

as:

$$(p(t))^2 = \sum_{i=0}^{6} c_i^2 t^{2i} + 2\sum_{i<j}\sum_{j=1}^{6} c_i c_j t^{i+j}$$

Equation 15

that polynomial is of order 12 and can be rewritten as

$$p^2(t) = c_0^2 + (2c_0c_1)t + (2c_0c_2 + c_1^2)t^2 + (2c_0c_3 + 2c_1c_2)t^3 + (2c_0c_4 + 2c_1c_3 + c_2^2)t^4 + (2c_0c_5 + 2c_1c_4 + 2c_2c_3)t^5 + (2c_0c_6 + 2c_1c_5 + 2c_2c_4 + c_3^2)t^6 + (2c_1c_6 + 2c_2c_5 + 2c_3c_4)t^7 + (2c_2c_6 + 2c_3c_5 + c_4^2)t^8 + (2c_3c_6 + 2c_4c_5)t^9 + (2c_4c_6 + c_5^2)t^{10} + (2c_5c_6)t^{11} + c_6^2t^{12} = p^*(t) = \sum_{i=0}^{12} k_i t^i$$

Equation 16

with the coefficients k_i defined in table 3

k_0	k_1	k_2	k_3	k_4	k_5	k_6
c_0^2	$2c_0c_1$	$2c_0c_2 + c_1^2$	$2c_0c_3 + 2c_1c_2$	$2c_0c_4 + 2c_1c_3 + c_2^2$	$2c_0c_5 + 2c_1c_4 + 2c_2c_3$	$2c_0c_6 + 2c_1c_5 + 2c_2c_4 + c_3^2$
k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	
$2c_1c_6 + 2c_2c_5$	$2c_2c_6 + 2c_3c_5 + c_4^2$	$2c_3c_6 + 2c_4c_5$	$2c_4c_6 + c_5^2$	$2c_5c_6$	c_6^2	

Table 3: The coefficients of the polynomial that calculates the quadratic difference between the approximations: I^4, I^6 It is a measure of how much they disagree with each other for a defined range of values.

1. In order to obtain the squared error of both models

Concept	Values	Formula
ihoradia	0,1,…,12	
horai	0,1,2,3,4,5,6,5,4,3,2,1,0	$horai = 6 - \|6 - ihoradia\|$
θhorai	$0,\frac{\theta_{max}}{6},..,\theta_{max},...,\frac{\theta_{max}}{6},0$	$\theta horai = \theta max * \frac{horai}{6}$
radianhorai	$0,\frac{radianhora_{max}}{6},...,\frac{radianhora_{max}}{1},...,0$	$radianhorai = \theta horai * \left(\frac{\pi}{180}\right)$
horadiaisuni		$horadiasuni = mod(\|i+6\|, 12)$
Horazenithal	$[horacenital_{mini},...,6, hourzenithal_{minf}]$	$horacenitali = \chi_{horai+}\frac{\theta horai}{\theta max} * 6 + \chi_{horai-}(12 - \frac{\theta horai}{\theta max} * 6)$

Table 1: The different concepts defined, as well as the calculation formula for these and some discrete values that they take [1].

Coefficients of the model polynomials of order 4 and 6 and difference in solar irradiation

a0	a1	a2	a3	a4		
-7.96336983	1.74003224	3.72E-11	-0.000107307	2.98E-07		
b0	b1	b2	b3	b4	b5	b6
-2.4456	1.0505	0.0247	-0.000511	3.80E-06	-1.58E-08	2.93E-11
a1-b0	a1-b1	a2-b2	a3-b3	a4-b4	-b5	-b6
-5.51776983	0.68953224	-2.47E-02	0.000403693	-3.50E-06	1.58E-08	-2.93E-11

Table 2: The coefficients: a_i for the polynomial of I^4, b_i for the polynomial I^6 nd $(a_i - b_i)$ of $I^4 - I^6$

1. In fact, these values can be extended to the continuum, but only the integer values are shown in the case of horai, ihoradia, horadiasuni, this can be done considering no longer the sexagesimal system but the decimal system. And the adjustment would be made to the sexagesimal by converting the fraction of horai to a fraction in sexagesimal minutes, with 0.5 hora equivalent to 30 minutes.

If we calculate the total squared difference as in Equation 17 and apply the fundamental theorem of calculus.

$$error_q = \int_{t_0}^{t_f} k_i t^i \, dt = \sum_{i=0}^{12} \left| \frac{k_i}{i+1} t^{i+1} \right|_{t_0}^{t_f} = \sum_{i=1}^{13} K_i (t^i{}_f - t_0^i)$$

Equation 17

where the parameters: are given by table 4

Table 4: The polynomial coefficients are the result of the calculation of the quadratic discrepancy between both models in the primitive range of the fitting data.

k_0	k_1	k_2	k_3	k_4	k_5	k_6
0	k_o	$\frac{k_1}{2}$	$\frac{k_2}{3}$	$\frac{k_3}{4}$	$\frac{k_4}{5}$	$\frac{k_5}{6}$
k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}
$\frac{k_6}{7}$	$\frac{k_7}{8}$	$\frac{k_8}{9}$	$\frac{k_9}{10}$	$\frac{k_{10}}{11}$	$\frac{k_{11}}{12}$	$\frac{k_{12}}{13}$

Table 4 the values of the coefficients of the polynomial p(t) and the integral of p(t)*

Note: Polynomials of order 4 and order 6: * $I^4 = \sum_{i=0}^{4} a_i t^i$; $I^6 = \sum_{i=0}^{6} b_i t^i$;

Note: quadratic error polynomial: $p(t) = (I^4 - I^6)^2 = \sum_{i=0}^{12} k_i t^i$;

Note: The integral of the quadratic error polynomial: $P(t) = \int_{t(\theta_{mini})}^{t(\theta_{minf})} p(t)dt = \sum_{i=0}^{13} K_i t^i$

CONCLUSIONS

The method for calculating hours in the day is proposed under the concept of local solar day in equation 10.

The irradiance equation for the day is developed under the assumption that it can be model by a polynomial of order four, and at the same time the adjustment is reported for the case of considering a polynomial of order 5.

The total radiation in which both polynomials differ is calculated by considering the integral of the difference between the values of the two polynomials and a calculation sequence is given for the coefficients of the generated quadratic error.

FINAL COMMENTS

Although it could be understanded that this methos of calculus of the day supposes the plane surface in reality it is locally supposed to be plane it is not in contradiction with the fact by the spherical form of the hearth but remember that for the static point of view the sphericity is not important and it is really unknown, a local observer dependent of the solar radiation only can sense the existence or absent solar radiation. For a moving observer of the solar radiation the theory of description of the moving observed and computed time.

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