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ESTIMATION OF THE EFFECTIVE EARTH RADIUS FACTOR (*K-FACTOR*) FOR PRECISE CALCULATION OF THE PATH PROFILE IN POINT-TO-POINT RADIO LINKS

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Abstract: The article addresses the estimation of the *K*-factor to achieve an accurate calculation of the radio path profile in line-of-sight (LOS) point-to-point radio links. The radio path profile is essential to ensure the elimination of obstacles within the first Fresnel ellipsoid, through the precise placement of the antennas high above the Earth's surface. This arrangement facilitates proper reception of the transmitted beams, which arrive at the receiving antenna from various paths, all synchronized in phase with the direct line-of-sight beam. Additionally, the impact of the Earth's atmosphere on radio frequency (RF) signals is analyzed, two approaches to calculate the *K*-factor are outlined, and the procedure for plotting the radio path profile is described.
Keywords: Radio path profile, LOS point-to-point radio link, first Fresnel ellipsoid, tower height for antennas.

INTRODUCTION

In the design of a microwave line-of-sight radio link, it's essential to accurately calculate the height position of the antennas relative to the Earth's surface. This ensures that the first Fresnel ellipsoid remains unobstructed, thereby ensuring proper signal reception at the transceiver equipment. It's also important to account for the curvature of the Earth and potential changes in the surrounding topography and environmental conditions.

Furthermore, the position of the antennas influences the length of the transmission lines connecting to the transceiver equipment. These lengths determine the loss in decibels of the RF signal power at the antenna input. It is essential that rays not aligned with the main lobe's maximum gain beam arrive in phase with the transmitted RF signal.

In reference [1], certain procedures are proposed to determine the optimal height of the antennas. These procedures focus on analyzing the clearance of obstacles from the first Fresnel ellipsoid, starting from the estimated value of the *K*-Factor, specifically for the temperate and tropical climates found in the radio link region.

The K factor may exhibit low values due to diffraction effects, leading to situations of sub-refractivity, a phenomenon predominant approximately 99.9% of the time. Furthermore, in relation to [2], it is essential to consider both the mean and effective values of the *K*-factor to determine the appropriate height of the antennas. This analysis is crucial to ensure clearance of the path of maximum gain beam of the antennas and prevent losses due to diffraction caused by fading resulting from sub-refractivity in the design of a specific microwave radio link.

To ensure optimal clearance of the path, we propose a methodology that involves obtaining the radio-electric path profile between the radio link sites. This profile should consider atmospheric refractivity conditions, using the minimum value of the effective *K*-factor along the wave propagation path. Based on this radio link profile, the appropriate height of the antennas is determined.

Similarly, atmospheric phenomena and their influence on RF signals are examined through the analysis of refractivity parameters. Two approaches are presented to calculate the *K*-factor, and the procedure for plotting the radio-electric path profile is described. The *K*-factor considers the curved propagation of electromagnetic waves in the Earth's atmosphere [3,4].

To simplify the study, the line-of-sight beam between the antennas is traced, following a straight trajectory, and considering the variation of the Earth's surface through the *K*-factor.

Finally, the methodology for creating the diagram of the radio-electric path profile between the two point-to-point link sites is detailed, starting with the real geographic profile from a topographic map at a scale of 1:50000 or from Google Earth®. This proposal is based on the lack of specialized software that has a database of specific contour lines for the region.

FREE SPACE PROPAGATION LOSS

Let's briefly imagine a scenario where there is no atmosphere, and the Earth is flat. Under such conditions, a beam emitted by an antenna will experience attenuation or energy loss towards another receiving antenna equal to [5]:

$$L_{FS} = (4\pi\, d\, f/c)^2 \tag{1a}$$

$$L_{FS,dB} = 92.4417 + 20\log \mathrm{d}_{Km} + 20\log f_{GHz} \tag{1b}$$

where d, f and c represent the distance between antennas, the carrier frequency, and the speed of wave propagation in free space, respectively. L_{FS} is also known as free space path loss. Additionally, the received RF signal is attenuated by multiple beams following different paths [6].

Because the Earth's surface is not flat, calculating the reflected beam is complicated. This beam can cause constructive interference (when it arrives in phase) or destructive interference (when it arrives out of phase) on the signal reaching the antenna with direct visibility. These phenomena result in selective fading, which is a critical factor in the quality of a digital radio link.

RADIUS OF THE CURVED BEAM PATH

During point-to-point line-of-sight connections, RF signals pass through the troposphere, encountering variations that lead to atmospheric refraction (AR). This phenomenon occurs when the refractive index of the atmosphere varies along the path of the ray, resulting in the curvature of signal propagation from one antenna to another [4].

A key question arises: What is the magnitude of the curvature experienced by the ray's path in the atmosphere? Since the atmosphere is composed of various layers, strata, or planes, each with a constant and different refractive index (n), ducts are formed through which the RF signal propagates during its transmission in free space [3].

Figure 1 represents the trajectory of the signal beam as it passes through a layer of the atmosphere. It shows the angle (φ) with respect to the normal at a point (c) of the layer, along with the height of the layer (dh) and the differential angle ($d\varphi$) formed by the curvature radii of the trajectory (R) of the beam at points a and b.

From the figure it is possible to establish:

$$R = \frac{\overline{ab}}{d\varphi} \;\; [\mathrm{m}] \tag{2}$$

At point b: $\cos(\varphi + d\varphi) = \frac{dh}{\overline{ab}}$ and considering that: $d\varphi \approx 0$:

$$\overline{ab} = \frac{dh}{\cos(\varphi)} \tag{3}$$

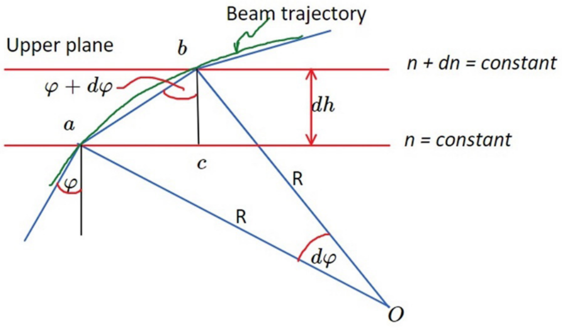

Fig. 1 Path of the signal beam through the atmosphere layer

From equations (2) and (3) we obtain:

$$R = \frac{dh}{d\varphi \cos(\varphi)} \tag{4}$$

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In a medium with a gradual variation of its refractive index, the law of refraction remains constant at each point along the propagation path, that is:

$$n \sin \varphi = (n + dn) \sin (\varphi + d\varphi)$$

And considering: $\cos(\varphi) \cong 1$ and $\sin(\varphi) \cong d\varphi$ is obtained.

$$d\varphi \cos(\varphi) = -\frac{dn}{n} \sin(\varphi) \tag{5}$$

Replacing Equation (5) into Equation (4) we arrive at:

$$R = \frac{dh}{-\frac{dn}{n} \sin(\varphi)} \tag{6}$$

Given that the refractive index (n) of air is approximately equal to 1 [3] and that the rays propagate at very low angles ($\sin(d\varphi) \cong 1$), equation (6) simplifies to:

$$R = \frac{1}{-\frac{dn}{dh}}$$

If $N = n \times 10^6$, therefore:

$$R = \frac{10^6}{-\frac{dN}{dh}} \tag{7}$$

In the lower troposphere, the curvature radius of the ray's trajectory (R) is defined by the rate of change of the refractive index with respect to height, rather than its absolute value. The negative sign in equation (7) indicates that R will be positive, implying that the propagation trajectory will be convex (when viewed from above) only when the refractive index (n) decreases with height. This is the typical scenario in a standard atmosphere.

In a standard atmosphere, it is commonly assumed [4] that $\frac{dn}{dh} = -4 \times 10^{-2}\ m^{-1}$. Using equation (7), the radius of curvature of the trajectory (R) is found to be equal to 25,000 km.

EFFECTIVE EARTH RADIUS FACTOR AS A FUNCTION OF BEAM CURVATURE RADIUS (R)

Both the direct ray and the reflected ray follow curved paths due to the influence of the atmosphere. The propagation speed in the lower troposphere, where the refractive index (n) values are high, will be lower than in the upper troposphere.

We consider two propagation cases. In the first one (case 1), the path of the ray is curved, meaning convex due to the presence of the atmosphere. In the second one (case 2), the path of the ray is straight, meaning without the influence of the atmosphere. To correct this, a larger Earth radius than the radius (a) is considered. In case 2, the curvature radius of the trajectory will be like the curvature defined by the effective Earth radius (a'). Figure 2 presents the established propagation cases.

The relative curvature of case 1 is: $\frac{1}{a} - \frac{1}{R}$.

The relative curvature of case 2 is: $\frac{1}{a'} - \frac{1}{\infty}$.

By equating the relative curves of both cases and solving for the effective Earth radius (a'), it is found that a' equals to:

$$a' = \frac{a}{1 - \frac{a}{R}} \ \text{[m]} \tag{8}$$

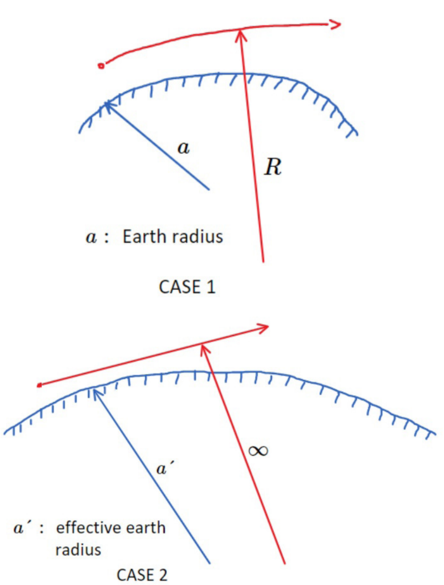

Fig. 2 Trajectories of ray propagation in the atmosphere

When substituting equation (7) into equation (8), the K-factor ($K = a'/_a$) turns out to be:

$$K = \frac{1}{1 + a\,\frac{dN}{dh}\,10^{-6}} \tag{9}$$

For example, with a standard refraction (in a standard atmosphere) of:

$$\frac{dN}{dh} = -4 \times 10^{-2}\, m^{-1}$$

and taking the Earth radius (a) as 6.73 x 10^6 meters (6370 km), the K-factor (according to Eq. (9)) turns out to be 1.341921 (≅ 4/3), resulting in an effective Earth radius (a') of 8548 km.

If we take the gradient of the refractive index (ΔN) as equal to: $\frac{dN}{dh}$ x 10^3, then the curvature radius of the ray's trajectory (R) is:

$$R = \frac{10^6}{-\Delta N \times 10^{-3}} \tag{10}$$

Substituting equation (10) into equation (9), we obtain:

$$K = \frac{1}{1 + a\,\Delta N\, \times 10^{-9}} \tag{11}$$

alternatively,

$$K = \frac{1}{1 - \frac{a}{R_{Km}}} \tag{12}$$

For example, in a geographic region with ΔN = -54 ($\frac{dN}{dh}$ = -0.054 m^{-1}), the value of the ray's trajectory curvature radius (R) is 18519 km. According to equation (12), the K-factor is equal to 1.5243.

RADIO SIGNAL PROPAGATION IN THE ATMOSPHERE

The electromagnetic waves associated with RF signals propagate through a dielectric medium with a refractive index close to unity, corresponding to the air in the troposphere. This atmospheric layer extends from the Earth's surface to approximately 8 to 10 kilometers in height at the poles and up to around 16 to 18 kilometers at the Equator. The troposphere is the main environment for atmospheric convection, where clouds form, and most precipitation occurs.

The refractive index of the troposphere for radio waves is expressed as $n = \sqrt{\varepsilon\mu}$, where ε is the relative permittivity and μ is the relative permeability. For air, this index varies around 1.0003, with variations typically found in the fifth and sixth decimal places.

For a better understanding, radio refractivity (N) is expressed in terms of the tropospheric refractive index (n), according to the following expression:

$$N = (n - 1) \cdot 10^6 \tag{13}$$

which can be approximated for a given region using the following equation:

$$N = 77.6\, P/T + 373256/T^2 \cdot a\, exp(bt/(t + c)) \tag{14}$$

with the knowledge of temperature (T), pressure (P), and the constants t, b, and c for water and ice according to [7-9].

The refractive index N also depends on the refractivity at the surface N_S (or radio-electric refractive index) up to the first kilometer of altitude, whose value decreases with height as established in [7].

$$N(h) = N_s exp(-c \cdot h) \tag{15}$$

where the variable: $c(=\ln(N_S/(N_S+\Delta N)))$ depends on the refractivity gradient ΔN, interpreted as the variations in refractivity in the N values.

According to equation (13) the radioelectric refractive index at the surface n_S will be:

$$n_s = 1 + N_s \times 10^{-6} \tag{16}$$

Similarly, the surface refractivity N_S is related to the negative gradient of the radioelectric refractive index ΔN, as established in [7]:

$$-\Delta N = 7.32\, \exp(0.005577 \cdot N_s) \tag{17}$$

Due to the atmospheric refractive index, electromagnetic beams deviate from their path. Additionally, radio waves experience absorption and dispersion along their journey, with intensity varying depending on frequency and altitude relative to sea level [10].

In the troposphere, the effects of weather changes such as rain, snow, and water vapor are evident. Temperature, pressure, and vapor content changes cause radio waves in the troposphere to undergo both refraction and dispersion.

Absorption and dispersion due to rain and snow are particularly noticeable in signals with frequencies above 10 GHz. Water vapor has a significant effect on signals with frequencies above 22 GHz. Weather variations cause absorption and dispersion of energy when signals interact with them, resulting in signal attenuation.

EFFECTIVE EARTH RADIUS FACTOR AS A FUNCTION OF SURFACE ATMOSPHERIC REFRACTIVITY (NS)

To facilitate the study of wave propagation between antennas, it is essential to relate various atmospheric conditions to the curvature of radio waves due to diffraction.

Under normal conditions, the trajectory of the beam curves according to the *K*-factor. This phenomenon is due to the gradient of the radioelectric refractive index (ΔN), which depends on the surface refractivity (N_S) and the altitude above sea level [11]. The *K*-factor can be estimated using the following equation:

$$K = \frac{1}{1 - \frac{a}{n_s} \cdot \ln\left(\frac{N_s}{N_s + \Delta N}\right) \cdot N_s \cdot 10^{-6}} \tag{18}$$

The ITU-R Recommendation P.453-10 from the International Telecommunication Union [7] provides curves showing the approximate values of the gradient ΔN for Mexico across various months of the year. Figure 3 depicts the negative values of the refractivity gradient (-ΔN) corresponding to the months of February, May, August, and November.

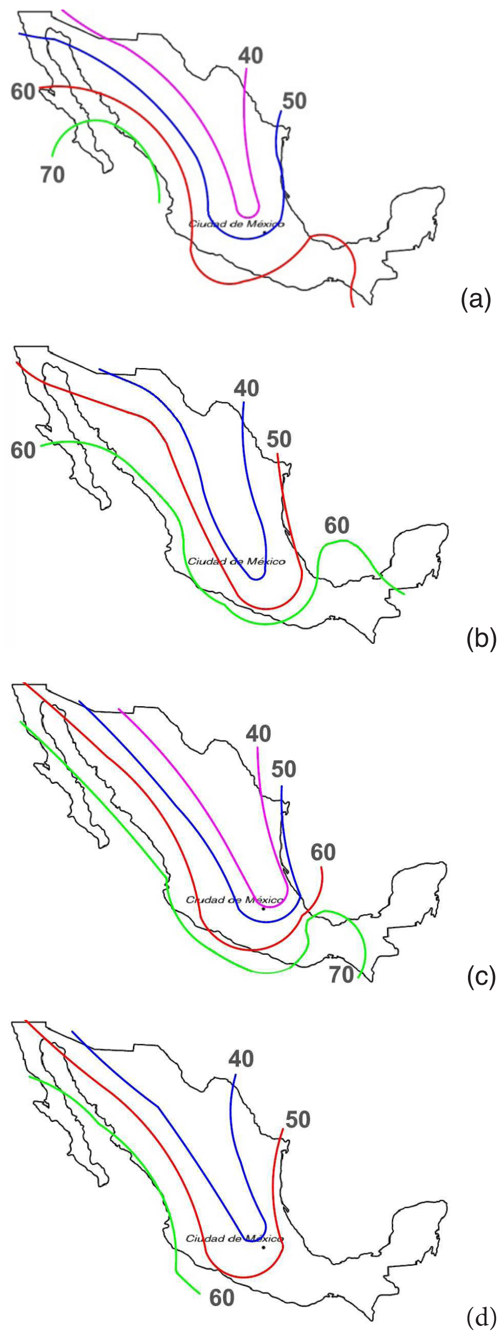

Fig. 3 Reduction in the gradient of the radioelectric refraction index (-ΔN) estimated by [7] for Mexico during the months of (a) February, (b) May, (c) August and (d) November.

For example, when designing a Line-of-Sight (LOS) radio link in a state of Mexico with a monthly average refractivity gradient ΔN=-50 (see Figure 3), we calculate the surface refractivity N_S using equation (17):

$$N_s = 179.31 \cdot \ln(|\Delta N/7.32|) = 344.53$$

Surface refractive index (n_S) is obtained from equation (16), and the *K*-factor to consider, according to equation (18), is *K*=1.5248.

Therefore, in this region, the curved trajectory of the beams is greater than that followed in an atmosphere characterized by a *K*-factor equal to 4/3.

The value of the *K*-factor is influenced by the signal frequency, specific geographic location, and atmospheric conditions at the time of transmission. Different values of K result in different propagation behaviors of RF signals, as they are directly related to the atmospheric gradient characteristics at the location where the Line-of-Sight (LOS) microwave radio link is designed.

Figure 4 illustrates the types of curvature experienced by the beams during their propagation in free space.

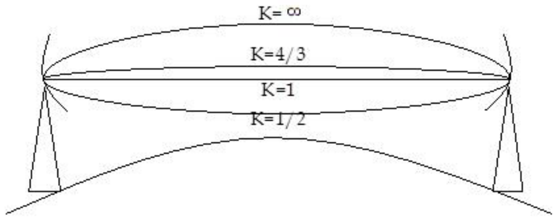

Fig. 4 Beam curvature types in various atmospheres

The *K*-factor occurs in 50% of cases. However, in Argentina, due to its climate, this factor is present in 60% of instances. When *K*<4/3 a substandard atmosphere is encountered. If *K*=1 the beam will follow a direct trajectory.

The substandard atmosphere is formed due to fog generated by the interaction between warm air and cold air, or over a moist surface. This results in lower density near the ground than at higher altitudes, causing the beam to curve upward. A *K*=0.8 is recorded only 1% of the time.

FRESNEL ELLIPSOIDS

The diffraction of electromagnetic signals in air particles results in what is known as ellipsoids or Fresnel zones. For study purposes, these Fresnel zones are considered a family of ellipsoids formed in the propagation medium through which RF signals travel from the transmitter to the receiver. A Fresnel zone is defined by the boundaries where internal waves reach the receiver with the same phase as the transmitted signal. Therefore, the phase of signals in Fresnel zones alternates as follows: in phase (first zone), out of phase (second zone), in phase (third zone), and so on.

The radius of the Fresnel zone depends on the wavelength (λ) and the distance between the antennas. For signals with low wavelengths, the difference between different paths results in a quicker out-of-phase zone, hence the Fresnel radius is smaller. If d_1 and d_2 represent the distances from the sites to the point where the Fresnel zone is to be determined, which is characteristic of the most prominent obstacle in the propagation path, the radius (in meters) of the first Fresnel zone is calculated using equation [1,5]:

$$r_f = 547\sqrt{\frac{d_1\, d_2}{f\, d}} \tag{19}$$

where $d(=d_1+d_2)$ represents the total distance of the link. The distances d_1, d_2 and d, are measured in ***Km*** and the carrier frequency in MHz.

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Alternatively, you can use the equation:

$$r_f = 17.32\sqrt{\frac{d_1\, d_2}{f\, d}} \quad (20)$$

distances d_1, d_2 and d are in Km and the carrier frequency in GHz. In practical applications, it is desired that the first Fresnel zone remains completely unobstructed.

DESIGN OF A TERRESTRIAL RADIO LINK PROFILE

The purpose of profiling the radio link, considering the influence of the atmosphere on radiofrequency signals, is to determine the optimal altitude for placing each of the antennas. These antennas will be mounted on metallic structures, facing each other, with the aim of ensuring that the first Fresnel ellipsoid remains unobstructed.

With knowledge of the heights of both antennas' towers (h_1 and h_2), we can establish the length of the antennas' feed lines and calculate the losses in the feeders (L_{FTX}, L_{FRX}) of the antennas. These losses are determined using data provided by the manufacturer regarding loss in decibels per every 100 meters on the RF signal. These feeder losses are added to the losses existing between the transmitter device's output and the receiver device's input, i.e., to the system gain [13].

As a result of the previously described methodology, Figure 5 presents an example of the radio path profile between sites A and B, detailing the heights H_1 and H_2 above sea level of each location. This radio link profile encompasses the tracing of the direct beam between the antennas of both sites with respect to sea level, the first Fresnel zone, and the heights of the antenna towers (h_1, h_2), respectively.

The deviation (h) of the topographic profile obtained from the map (L_M) generates the curve defined by (h_t). This deviation is calculated using the following equation:

$$h = d_1 d_2 / 12.75 / \mathrm{K} \quad (21)$$

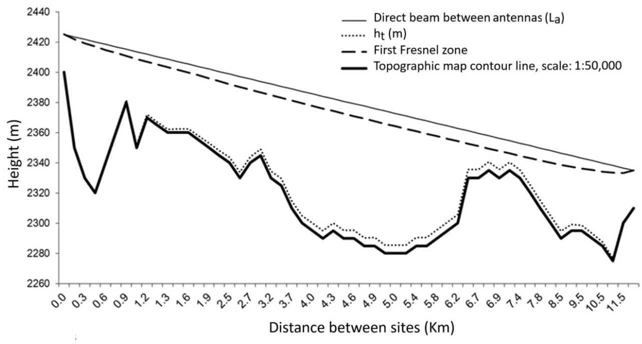

Fig. 5 A 12 km point-to-point radio link in line of sight (LOS)

The design parameters of the radioelectric profile are specified in Table 1.

Parameter	Description
L_M	Topographic profile obtained from a topographic map with a scale of 1:50,000 or from Gooble Earth ®.
h	Earth profile deviation.
h_t	Height of the modified topographic profile (with K-factor) respect to sea level.
L_a	Height of the direct beam relative to sea level.
r_f	Radius of the first Fresnel ellipsoid relative to sea level.
e	Height from the contour line with factor K to the direct beam.

Table 1. Radioelectric path profile parameters

The following parameter is used to trace the direct beam between the two antennas with direct visibility:

$$i = |H_A - H_B| / (N_d - 1) \quad (22)$$

Where N_d represents the number of contour lines (or data points) between the antennas of both sites, obtained from the topographic map, that intersect the beam with direct visibility. $H_A(H_1+h_1)$ and $H_B(H_2+h_2)$ indicate the heights of the antennas relative to sea level.

To calculate the distance of the direct beam (L_a) with respect to sea level, the parameter i is added to each previous value of L_a if H_A is less than H_B, or subtracted if H_A is greater than H_B.

The distance from the end of the first Fresnel ellipsoid to sea level (Z_f) is calculated as the difference between the height of the direct beam (L_a) and the radius of the first Fresnel ellipsoid (r_f) at each point of the contour line data, i.e., .

$$Z_f = L_a - r_f.$$

The region between the first Fresnel ellipsoid and the Earth's profile with *K*-factor (h_t) is known as the clearance zone. Clearance (e) is ensured if the distance from the direct beam path to the most pronounced obstacle on the contour line, defined by h_t is greater than or equal to the radius of the first Fresnel zone at that point.

BUILDING THE RADIO PATH PROFILE

In a spreadsheet, a table is created for the radio link profile similar to Figure 5, organizing the parameters with their values in columns: N_d, d_1(Km), d_2(Km), L_M(m), L_a(m), h(m), h_t(m), r_f(m), Z_f(m), e(m) and e/r_f.

The parameters, such as h_1, h_2, H_1, H_2, *i*, *K*, *f* and *d*, are organized in individual cells, each with a single value. These cells are connected through the corresponding columns of the previous parameters.

Using the image library of the spreadsheet, it is possible to create a graph representing the radioelectric path profile. This graph will display the values of the columns L_M, L_a, h_t, and Z_f on the Y-axis, as a function of d_1 on the X-axis.

If all values in the e/Z_f column are greater than one ($e>r_f$), then it ensures that the first Fresnel ellipse is unobstructed, thus ensuring proper transmission.

CONCLUSIONS

A methodology is presented for the optimal design of the radio path profile in a point-to-point line-of-sight link. The profile, developed in a spreadsheet, allows for accurately calculating the appropriate height of the antennas to ensure clearance of the first Fresnel ellipsoid. This ensures the reception of the radiofrequency signal at the microwave transceivers at both ends of the link. The method considers the curved propagation of radiofrequency signals in the Earth's atmosphere, accurately calculating the effective Earth radius factor.

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