v. 4, n. 12, 2024

Journal of
Engineering
Research

RELATIVISTIC KINEMATICS IN ANISOTROPIC SPACES

Alvaro Saucedo
Estudiante de Maestría en Ciencias Físicas, SUE Caribe, Universidad del Atlántico
Puerto Colombia de Barranquilla, Colombia

Ubaldo e Molina Redondo
Mg. En Ciencias Físicas, Profesor del Departamento de Física, Universidad del Atlántico
Puerto Colombia de Barranquilla, Colombia
https://orcid.org/0000-0002-3537-6636

Ingrid Steffanell
Dr. En Ciencias Técnicas. Docente de la Facultad de Ingeniería, Universidad Libre de Barranquilla, Colombia
https://orcid.org/0000-0003-1216-9922

Abstract: A study of the Lorentz coordinate transformations is made accepting the validity of the two Einstein postulates and the spatial-temporal homogeneity, but in space-time lacking isotropy. Expressions relating the space-time coordinates between two inertial observers are found. Under the new equations of coordinate transformations, a study is made of the way transformation equations take for both velocity and acceleration measured by two inertial observers. These transformations are analyzed to see which ones remain unchanged, which change and which form change with respect to those obtained from spatial relativity in isotropic spaces. The invariance of the space-time interval is also studied.
Keywords: Anisotropic space-time, coordinate transformations, space-time interval, velocity transformations.

INTRODUCTION

Einstein's special relativity is based on his two postulates such as the invariance of the laws of physics measured in any inertial frame of reference and the postulate of the constancy of the speed of light regardless of the frame in which it is measured. In addition to these postulates is the additional ingredient to be added which consists of space-time isotropy. The basis that "the main idea of special theory" is that "there are no preferred frames of reference and therefore no preferred directions", which unduly confuses the isotropy of the unidirectional speed of light with the isotropy of space [1].

Classical books introduce it of isotropy (without even naming it) as a simple subject that does not need to be argued and that surely it is too hasty to make explanations of the spatial isotropy [2]. It is possible that spatial isotropy can be empirically proven, regardless of what is thought of the unidirectional speed of light, is one of the objectives of this work, which aims to offer a logically transparent path to anisotropic special relativity.

The idea in this research is to elaborate the assumption of anisotropy by exploiting the concept of an objective spatial direction in space-time compared to that elaborated by Minkowski, and use it in a very natural way to select some fundamental amounts of Physics and see the form they take when part of an anisotropic spacetime. It is possible to study how the Lorentz metric is replaced in non-isotropic spaces and to see the extent to which it tends to transform in the case of spatial isotropy [3].

It is then a question of studying the form that take fundamental themes of special relativity as they are: Lorentz transformations in coordinates, speed and acceleration when taking into account the spatial direction, in addition to the space-intervaltime, temporal dilation and contraction in length. Most specialized texts on this topic on special relativity do not consider it necessary to explain that space isotropy is not a trivial or necessary ingredient in the principle of relativity, but in our case the special anisotropic transformation and some of its consequences are explained [4].

COORDINATE TRANSFORMATIONS IN ANISOTROPIC SPACES

Taking into account the homogeneity in space-time, the principle of relativity and the principle of the constancy of the speed of light, but not the space-time isotropy, the Lorentz transformation equations have the form [5]:

$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \alpha(t - \epsilon x) \end{aligned}\right\} \quad (1)$$

Assuming the constancy of the speed of light for an observer in system S, any signal emitted from the origin and traveling in the direction of the axis (+x) you get that, $x = ct$.

Replacing in equations (1) you get that,

$$\left.\begin{array}{l} x' = \gamma(ct - vt) = \gamma(c - v)t \\ t' = \alpha(t - c\epsilon t) = \alpha(1 - c\epsilon)t \end{array}\right\} \quad (2)$$

By the principle of constancy of the speed of light, for an observer in the S' system; the signal emitted from the origin in the direction of the axis ($+x'$), you have to $x' = ct'$. By replacing in the first equation (2) and multiplying the second equation (2) by c and matching these two, you get,

$$\gamma(c - v) = c\alpha\,(1 - c\epsilon) \quad (3)$$

Proceeding analogously with equations (1), but assuming that the direction of propagation of the emitted light beam occurs in the direction of the axis ($-x$), then $x = -ct$ and also in the direction of axis ($+x'$), that is, $x' = -ct'$.

This procedure allows the expression to be obtained,

$$\gamma(c + v) = c\alpha\,(1 + c\epsilon) \quad (4)$$

Adding member to member equations (3) and (4) is obtained,

$$2\gamma c = 2c\alpha\ ; \quad \alpha = \gamma$$

Subtracting member to member equations (3) and (4) is reached,

$$2\gamma v = 2c^2\alpha\epsilon\ ;\ c^2\epsilon = v\ ;\ \epsilon = \frac{v}{c^2}$$

Lorentz transformations are expressed in form [5],

$$\left.\begin{array}{l} x' = \gamma(x - vt) \\ t' = \gamma(t - {}^{vx}/_{c^2}) \end{array}\right\} \quad (5)$$

parameter γ should depend on the relative speed of reference frames v.

To find a suitable ratio for non-isotropic space-time, a parameter φ is introduced, differentiating the hyperbolic relations $senh\varphi$ sen φ, $cosh\varphi$ and $tan\varphi$.

Be

$$\left.\begin{array}{r} \frac{v}{c} = \tanh\varphi \\ \gamma = g(\varphi)cosh\varphi \\ \gamma\frac{v}{c} = g(\varphi)senh\varphi \end{array}\right\} \quad (6)$$

By introducing these relations in the transformation equations (5) it remains:

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = g(\varphi)\begin{pmatrix} \cosh\varphi & -\mathrm{senh}\varphi \\ -\mathrm{senh}\varphi & \cosh\varphi \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} \quad (7)$$

Since γ is a constant parameter that depends basically on the relative constant velocity v of the frame S' relative to S, then the parameter φ also depends on v.

In this sense the function $g(\varphi)$ will also depend on the velocity v and the transformation equations (7), it is observed that when $g(y) = 1$ the Lorentz transformation in isotropic spacetime is obtained.

By defining the function,

$$g(\varphi) = e^{2n\varphi} \quad (8)$$

Where n is a constant dimensionless parameter. It follows from the definition (8) that the conditions $g(0) = g(\varphi) = 1$ are met, see [5].

The relation (8) is the most general possible solution, which leads to finding the following relations that arise from equation (6),

$$\left.\begin{array}{r} \gamma = g(\varphi)cosh\varphi = \left(\frac{c+v}{c-v}\right)^n \frac{1}{\sqrt{1-{}^{v^2}/_{c^2}}} \\ \frac{v}{c}\gamma = g(\varphi)senh\varphi = \frac{v}{c}\left(\frac{c+v}{c-v}\right)^n \frac{1}{\sqrt{1-{}^{v^2}/_{c^2}}} \\ tanh\varphi = \frac{v}{c} \end{array}\right\} \quad (9)$$

Taking into account the above equations (9), the transformation equations (7) remain as follows,

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \left(\frac{c+v}{c-v}\right)^n \frac{1}{\sqrt{1-{}^{v^2}/_{c^2}}}\begin{pmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}$$

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \gamma(n, v)\begin{pmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} \quad (10)$$

Being in this case the relativistic factor in anisotropic spaces $\gamma(n, v)$, which is defined in the form,

$$\gamma(n,v) = \left(\frac{c+v}{c-v}\right)^n \frac{1}{\sqrt{1-v^2/c^2}} \tag{11}$$

For the case where the dimensionless number $n = 0$, the relativistic factor of special relativity is obtained in isotropic spaces, i.e.,

$$\gamma(0,v) = \frac{1}{\sqrt{1-v^2/c^2}} = \gamma_0 \tag{12}$$

It is now appropriate to find the remaining transformations between the directions of axis yy' and axis zz'. Suppose first that a ray of light moves in the xy plane for the observer S and in the $x'y'$ plane for the observer S', so that the following relations are given for the ray of light,

$$\left.\begin{array}{l}\text{Plano xy:}\quad x^2+y^2=c^2t^2\\ \text{Plano } x'y' \quad x'^2+y'^2=c^2t'^2\end{array}\right\} \tag{13}$$

Using the relations of equation (10), which can be written, $ct' = y(n,v)\left(ct - \frac{vx}{c}\right)$ and $x' = y(n,u)(x-ut)$, replacing them in the second relation of equation (13) and clearing y' is obtained,

$$y' = \left(\frac{c+v}{c-v}\right)^n y \tag{14}$$

Similarly lies the transformation of zz' axes.

In short, the transformation of coordinates into anisotropic spacetime in the form of a matrix is thus [6],

$$\begin{pmatrix}X'^0\\X'^1\\X'^2\\X'^3\end{pmatrix} = \begin{pmatrix}\left(\frac{c+v}{c-v}\right)^n\gamma_0 & -\left(\frac{c+v}{c-v}\right)^n\gamma_0\beta & 0 & 0\\ -\left(\frac{c+v}{c-v}\right)^n\gamma_0\beta & \left(\frac{c+v}{c-v}\right)^n & 0 & 0\\ 0 & 0 & \left(\frac{c+v}{c-v}\right)^n & 0\\ 0 & 0 & 0 & \left(\frac{c+v}{c-v}\right)^n\end{pmatrix}\begin{pmatrix}X^0\\X^1\\X^2\\X^3\end{pmatrix}$$

$$\begin{pmatrix}X'^0\\X'^1\\X'^2\\X'^3\end{pmatrix} = \left(\frac{c+v}{c-v}\right)^n\begin{pmatrix}\gamma_0 & -\gamma_0\beta & 0 & 0\\ -\gamma_0\beta & \gamma_0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}X^0\\X^1\\X^2\\X^3\end{pmatrix} \tag{15}$$

Which can be written in the simplest form,

$$x'^\alpha = \left(\frac{c+v}{c-v}\right)^n \Lambda_v{}^\alpha x^v \tag{16}$$

Where: α, u = 0,1,2,3. In addition to the notation changes: $X'^0 = ct'$, $X'^1 = x'$, $X'^2 = y'$, $X'^3 = z'$ for the primate system and $X^0 = ct$, $X^1 = x$, $X^2 = y$, $X^3 = z$ for the non- primate system.

In addition: γ_0 the relativistic factor, β dimensionless parameter and $\Lambda_u{}^\alpha$ is the Lorentz matrix of Einstein's special relativity (isotropic), defined as,

$$\gamma_0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \beta = \frac{v}{c} \text{ y } (\Lambda_v{}^\alpha) = \begin{pmatrix}\gamma_0 & -\gamma_0\beta & 0 & 0\\ -\gamma_0\beta & \gamma_0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \tag{17}$$

The previous transformations of coordinates represented in equations (15) are to pass from system S to system S' (S → S').

On the other hand, if we were to exchange our reference systems or what is the same consider that the coordinates given for the space and time of the event are those observed in S', instead of in S; the only change allowed by the principle of relativity would be the physical change in relative velocity of u by $-u$. That is, from S'; the S system moves to the left, while from S the S' system moves to the right [7].

CONSEQUENCES OF COORDINATE TRANSFORMATIONS IN ANISOTROPIC SPACES

This section discusses the consequences of coordinate transformations represented in equations (15). Basically, the study focuses on investigating the invariance or otherwise of the space-time interval, the form that velocity transformations take in anisotropic space as well as the form of accelerations transformations. It is verified if these transformations undergo fundamental changes compared to those of special relativity accepted in the literature for isotropic time-spaces.

SPACETIME INTERVALS IN ANISOTROPIC SPACES

When it comes to Einstein's spatial relativity with its postulates and properties of homogeneity and isotropy in both space and time, the space-time interval is an invariant quantity, that is, no matter the inertial observer of reference that measures them, the measured value remains unchanged,

$$dS'^2 = dS^2 \tag{18}$$

From these properties of spacetime and taking into account the definition of proper time τ for an observer in the S system ($dS = cd\tau$), and analogously for an observer in the S system ($dS' = cd\tau'$). When considering identity (18) and definitions of proper time, it is observed that,

$$d\tau' = d\tau \tag{19}$$

According to this result, one's own time is that which measures an observer at rest to the observed event or event.

As can be seen from equation (18) and from the Lorentz coordinate transformation equations the space-time interval in spatial relativity with its two postulates and the properties of space- time homogeneity and isotropy, is a relativistic invariant.

As will be seen in the next development, this space-time interval is not a relativistic invariant to the transformations of coordinates into anisotropic space-time. To do this, we develop dS'^2 using the transformations (15), which can be written as,

$$X'^0 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (\gamma_0 X^0 - \gamma_0 \beta X^1) = \gamma_0 \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (X^0 - \beta X^1)$$

$$X'^1 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (-\gamma_0 \beta X^0 + \gamma_0 X^1) = \gamma_0 \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (-\beta X^0 + X^1)$$

$$X'^2 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^n X^2$$

$$X'^3 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^n X^3$$

Where: $\gamma_0 = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$, $\beta = \frac{\upsilon}{c}$

The space-time interval according to an observer in the S' system is,

$$dS'^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$
$$= (dX'^0)^2 - \left(dX'^1\right)^2 - \left(dX'^2\right)^2 - (dX'^3)^2$$

Replacing and developing you have,

$$dS'^2 = \gamma_0^2 \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} (dX^0 - \beta dX^1)^2 - \gamma_0^2 \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} (-\beta dX^0 + dX^1)^2 - \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} (dX^2)^2 - \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} (dX^3)^2$$

$$dS'^2 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} \{\gamma_0^2[(dX^0 - \beta dX^1)^2 - (-\beta dX^0 + dX^1)^2] - (dX^2)^2 - (dX^3)^2\}$$

$$= \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} \{\gamma_0^2[(dX^0)^2 - 2\beta dX^0 dX^1 + \beta^2 (dX^1)^2 - \beta^2 (dX^1)^2 + 2\beta dX^0 dX^1 - (dX^1)^2] - (dX^2)^2 - (dX^3)^2\}$$

$$= \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} \{\gamma_0^2[(1-\beta^2)(dX^0)^2 - (1-\beta^2)(dX^1)^2] - (dX^2)^2 - (dX^3)^2\}$$

$$= \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2\{n\}} [(dX^0)^2 - (dX^1)^2 - (dX^2)^2 - (dX^3)^2]$$

Finally the relationship remains,

$$dS'^2 = \left(\frac{c+\upsilon}{c-\upsilon}\right)^{2n} dS^2 \tag{20}$$

As can be seen from the expression (20), the space-time interval is not relativistic invariant to transformations of type coordinates (15).

TRANSFORMATION OF SPEED INTO ANISOTROPIC SPACETIME

Let $\vec{u} = \frac{d\vec{r}}{dt} = u_x \hat{\imath} + u_y \hat{\jmath} + u_z \hat{k}$, the velocity of a particle in a reference system S and $\vec{u}' = \frac{d\vec{r}'}{dt'} = u'_x \hat{\imath} + u'_y \hat{\jmath} + u'_z \hat{k}$, the velocity measured by an observer in the system S', moving with constant speed υ relative to S in the direction of the xx' axes,

$$u'_x = \frac{dx'}{dt'} = \frac{cdX'^1}{dX'^0} = \frac{c\gamma_0 \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (dX^1 - \beta dX^0)}{\gamma_0 \left(\frac{c+\upsilon}{c-\upsilon}\right)^n (dX^0 - \beta dX^1)}$$

$$= \frac{c(dx - \beta(cdt))}{cdt - \beta dx} = \frac{c\left(\frac{dx}{dt} - \upsilon \frac{dt}{dt}\right)}{c\frac{dt}{dt} - \frac{\upsilon}{c}\frac{dx}{dt}}$$

Taking into account the definition of speed you get to the ratio of added speeds, that is,

$$u'_x = \frac{u_x - \upsilon}{1 - \frac{\upsilon u_x}{c^2}} \tag{21}$$

As can be seen, it does not undergo changes with respect to the transformation into isotropic spaces. Now, the analysis is done for the transformation of speeds on yy' and zz' axes,

$$u'_y = \frac{dy'}{dt'} = \frac{cdX'^2}{dX^0} = \frac{c\left(\frac{c+v}{c-v}\right)^n dX^2}{\gamma_0\left(\frac{c+v}{c-v}\right)^n (dX^0 - \beta dX^1)}$$

$$= \frac{c\, {dy}/{dt}}{\gamma_0\left(c\frac{dt}{dt} - \beta\frac{dx}{dt}\right)} = \frac{\mu_y}{\gamma_0\left(1 - \frac{v}{c^2}u_x\right)}$$

You get the speed transformation for the yy' axes, ie,

$$u'_y = \frac{\mu_y}{\gamma_0\left(1-\frac{v}{c^2}u_x\right)} \quad (22)$$

Proceeding similarly for the zz' axes, you get,

$$u'_z = \frac{u_z}{\gamma_0\left(1-\frac{v}{c^2}u_x\right)} \quad (23)$$

The results of the velocity transformation equations (21), (22) and (23) for anisotropic spaces are not altered compared to those found in isotropic spaces.

TRANSFORMATION OF ACCELERATION INTO SPACE-TIME ANISOTROPIC

Given an observer in system S that measures acceleration,

$$\vec{a} = \frac{d\vec{u}}{dt} = \frac{du_x}{dt}\hat{\imath} + \frac{du_y}{dt}\hat{\jmath} + \frac{du_z}{dt}\hat{k} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

While an observer in system S' moving with velocity u with respect to S, measuring acceleration,

$$\vec{a}' = \frac{d\vec{u}'}{dt'} = \frac{du'_x}{dt'}\hat{\imath} + \frac{du'_y}{dt'}\hat{\jmath} + \frac{du'_z}{dt'}\hat{k} = a'_x\hat{\imath} + a'_y\hat{\jmath} + a'_z\hat{k}$$

Developing the acceleration according to the x' axis measured by an observer in the S' system that moves with constant velocity u with respect to S,

$$a'_x = \frac{du'_x}{dt'} = \frac{cdu'_x}{dX'^0} = \frac{cd\left[\frac{u_x - v}{1 - {vu_x}/{c^2}}\right]}{\gamma_0\left(\frac{c+v}{c-v}\right)^n (dX^0 - \beta dX^1)}$$

$$= \frac{d\left[\frac{u_x - v}{1 - {vu_x}/{c^2}}\right]}{\gamma_0\left(\frac{c+v}{c-v}\right)^n \left[dt - \frac{v}{c^2}dx\right]}$$

$$= \frac{\left(1 - {vu_x}/{c^2}\right)(du_x) - (u_x - v)\left(-\frac{v}{c^2}\right)(du_x)}{\gamma_0\left(\frac{c+v}{c-v}\right)^n \left(1 - {vu_x}/{c^2}\right)^2 \left(dt - \frac{v}{c^2}dx\right)}$$

$$= \frac{\left(1 - {vu_x}/{c^2}\right)\left(\frac{du_x}{dt}\right) + (\mu_x - v)\frac{v}{c^2}\left(\frac{du_x}{dt}\right)}{\gamma_0\left(\frac{c+v}{c-v}\right)^n \left(1 - {vu_x}/{c^2}\right)^2 \left(1 - \frac{v}{c^2}u_x\right)}$$

$$= \frac{\left(1 - {vu_x}/{c^2} - {vu_x}/{c^2} - \frac{v}{c^2}\right)a_x}{\gamma_0\left(\frac{c+v}{c-v}\right)^n \left(1 - \frac{vu_x}{c^2}\right)^3}$$

By rearranging the equation the acceleration transformation equation for inertial observers is,

$$a_x{}' = \frac{1}{\left(\frac{c+v}{c-v}\right)^n}\frac{a_x}{\gamma_0^3\left(1-\frac{vu_x}{c^2}\right)^3} \quad (24)$$

It can be observed that the transformation of acceleration represented in equation (24) differs from that found in isotropic spaces in the factor: $1/\left(\frac{c+v}{c-v}\right)^n$

Following similar arguments, it can be seen that the transformations of the components of $a_y{}'$ and $a_z{}'$ accelerations measured in the S' system, in terms of the components in the S system in anisotropic time-spaces, are affected by the factor: $1/\left(\frac{c+v}{c-v}\right)^n$, compared to that found in isotropic spaces.

CONCLUSIONS

Traditionally the works and articles on special relativity are based on their original structure as is the principle of relativity for physics in general and that of the constancy of the speed of light in vacuum, in addition to the properties of homogeneity and spatiotemporal isotropy. In our case the study takes into account a spatial and temporal anisotropy, following basically the works of H. F. Goenner and G. Yu. Bogoslovsky in his article: "A Class of Anisotropic (Finsler) Space-time Geometries" [8], in which he finds the translations of Lorentz coordinates into anisotropic spaces, while in the works of Marco Mamone Capria in the article: "Spatial Directions, Anisotropy and Special Relativity" studies the concept of an objective spatial direction in special relativity and accepts the existence of a privileged spatial direction [6].

This extends the results of Goenner and G. Yu. Bogoslovsky, and from the set of Lorentz coordinate transformations are obtained the fundamental consequences that are derived from them, such as, in the part of kinematics the transformations of speed and acceleration, the invariance or otherwise of the space-time interval. From the results found it is evident that in space-time anisotropic velocity transformations do not undergo any change compared to those accepted of special relativity, while acceleration transformations, to switch from system S to system S', are affected by the factor: $1/\left(\frac{c+v}{c-v}\right)^n$, which makes it a different result than that obtained in the literature of special relativity. In the same sense the space-time interval according to equation (20), is affected by the factor: $1/\left(\frac{c+v}{c-v}\right)^n$, which does not allow it to be invariant to transformations (15).

In Einstein's spatial relativity the space-intervaltime is an invariant this makes proper time the same for any initial reference observer so no matter who measures the proper time of a physical event this is the same for any observer in both S' and S', but for a space- Nonisotropic time is not a relativistic invariant, one's own time depends on the observer who measures it.

It becomes interesting to generalize more mechanics in anisotropic spacetime, perhaps even contemplating future situations in which space and time are not homogeneous. It is believed that it could propose a more sophisticated theory of the structure of space-time.

REFERENCES

[1]Allais, M. L'anisotropie de l'espace. La nécessaire révision de certains postulats des théories contemporaines, Paris, Clément Juglar 1997.

[2]Arcidiacono, G. "A New 'Projective Relativity' Based on the De Sitter Universe", Gen. Rel. Grav., 7, 1976, 885-9

[3]Bogoslovsky G.Yu., Some physical displays of the space anisotropy relevant to the feasibility of its being detected at a laboratory, arXiv:0706.2621.

[4]Bogoslovsky G.Yu., Rapidities and Observable 3-Velocities in the Flat Finslerian Event Space with Entirely Broken 3D Isotropy, arxiv.org/abs/0712.1718v2.

[5]Drory, A., The necessity of the second postulate in special relativity. Studies in History and Philosophy of Modern Physics 51 (2015) 57-67.

[6]Mamone-Capria, M. "Spatial Directions, Anisotropy and Special Relativity". Cornell University. *Foundations of Physics* volume 41, pages1375–1397 (2011).

[7]Robert, R. "Introducción a la Teoría Especial de la Relatividad", Limusa. España, 1996.

[8]Goenner, H. F. and Bogoslovsky G.Yu., "A Class of Anisotropic (Finsler) Space-time Geometries", Preprint NPI MSU — 96–35/442, Moscow 1996. arXiv:gr-qc/9701067.