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INTERPOLATION USING CLASSICAL METHODS, A NEW METHOD IS OBTAINED USING CENTRAL DIFFERENCES, THEY ARE COMPARED WITH JASHIM'SUDDIN

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Abstract: The purpose of this article is to analyze the results of interpolation for a specific problem to decide, depending on the numerical result, which of these is closest to the real value. For this, the methods will be used: Newton's finite differences, Lagrange, Newton's divided differences, this new method that uses central differences and Jashim Uddin's:
Keywords: interpolation, finite differences, Newton, Lagrange, divided differences, central finite differences, Jashim Uddin

INTRODUCTION

The problem of approximating a quantity given a series of coordinates from data obtained from an experiment or a process is one of the oldest problems facing mathematicians. Its increasing importance in mathematics is used given (x_i, y_i)In much of the exact sciences, economics, biological sciences, medicine, social sciences, administration, it has a wide field of use. (Malik Saad Al-Muhja et al., 2019) in his article makes history of interpolation, (Perez Dilcia. et al, 2018) states the approximation theorem that was introduced by Weierstrass Stone. in 1985, this is based on a set of orthogonal continuous functions in which a polynomial of degree that approximates the proposed function is obtained. In this polynomial in which a point that is within. $nx_0<x_1...<x_n$

This theorem is described as follows.

Theorem (Weierstrassstone). continuous dicein f: [a, b] → $\mathbb{R}$in a compact interval, class $C_{[a,b]}$, given there exists an algebraic polynomial of degree, given by such that $\varepsilon>0$ $np_n(x)$ ll$f(x)$-$p_n(x)$ll<εA demonstration is found in the article by (DUNHAMJACKSONA., May, 1934) another demonstration of this theorem is due to Serge Bernstein in 1911 developed by (Matt Young 2006) he defines some polynomials that bear his name (Bernstein polynomials) this set of polynomials is given by, $B_n(x,f)$=

$\sum_{k=0}^{n} f\left(\frac{k}{n}\right) x^k (1-k)^{n-k}$,

$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!} \\ 0 \ in\ another\ case \end{cases}$ He is a linear operator.

This easily obtains the following using the following sequence of orthogonal polynomials which are $B_n(f)x = \frac{1}{(b-a)^n} \sum_{k=0}^{n} f\left((b-a)\frac{k}{n} + a\right)(x-a)^k (b-x)^{n-k}$

Which is used to prove the approximation theorem Weierstrass Stone. This series of polynomials form an orthogonal set on the interval, if generalized to an interval this can be transformed using a relation. This generalizes the theorem to find the interpolation polynomial. [0,1][a, b]t(x) = a + (b – a)x, x ∈ [0,1]

METHODOLOGY OR DEVELOPMENT.

The methods of Newton's finite differences, Lagrange, Newton's divided differences, central difference and Jashim Uddin will be used, comparing the results.

Firstly, the formation of the Newton interpolation method used by (Biswajit Das and Dhritikesh Chakrabarty 2018) to approximate values, also called Newton polynomial, will be given. In this type of interpolation, the finite differences are first obtained, the abscissa required for This means that the given values are equally spaced, the finite differences are obtained from the points where $x_{i+1} - x_i = cte(x_0, y_0), \ldots\ldots, (x_n, y_n) x_0 <$ $x_1 < \ldots x_0 < x_n$.

Using these points, the first difference is obtained by equation (1)

$$\Delta y = y_{i+1} - y_i \ (1)$$

The second difference by equation (2) and in equation (3) the nth forward finite difference (progressive)

$$\Delta^2 y = \Delta(\Delta y) = \Delta y_{i+1} - \Delta y_i (2)$$

$$\Delta^n y = \Delta(\Delta^{n-1} y) = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i (3)$$

In tabular form they are represented in table 1. For 4 points.

x_i	y_i	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$y_1 - y_0$		
x_1	y_1		$y_0 - 2y_1 + y_2$	
		$y_2 - y_1$		$y_0 - 3y_2 + y_3$
x_2	y_2		$y_1 - 2y_2 + y_3$	
		$y_3 - y_2$		
x_3	y_3			

Table 1. Symbolic obtaining of Newton's finite differences

The Newton polynomial is defined using forward finite differences. The polynomial is given as follows:

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + a_n(x-x_0)\ldots(x-x_{n-1})(4)$$

Where they are obtained in the following way. a_i

$$a_i = \frac{\Delta^i y_k}{i! \ \ h^i}(5) i = 0,1,\ldots,n$$

Lagrange Interpolation Polynomial. (Richard L. Burden and J. Douglas Faires. 2010 pp 108-114) develops the Lagrange interpolation polynomial method. In this case the values do not necessarily have to have the same spacing. This is represented in a polynomial of degree n, in general form given by equation (6) x_{i+1}-x_i

$$P_n(x) = y_0 L_0^n(x) + y_1 L_1^n(x) + \cdots, + y_n L_n^n(x) = \sum_{i=0}^{n} y_i L_i^n(x)(6)$$

Where the $L^n_i(x)$ are functions that depend on these are called Lagrange coefficients, which are defined by equation (7)x

$$L_i^n(x) = \prod_{j=0\ \ j\neq i}^{n} \left(\frac{\text{x-x}_j}{x_i\text{-x}_j}\right)^n \ (7)$$

This polynomial of the nth degree of Lagrange los,,...., coincides with the function for all $P_n\ x_0 x_1 f(x_i) = y_i i = 1, \ldots, 2$

Interpolation of Newton's Divided Differences. Using the book (richard l. burden and j. douglas faires. 2010 pp 123-133) where this method is also used when the values are equally spaced, but its use is for information that is not equally spaced. Suppose it is the nth polynomial of degree nth, the function with respect to,,....,. The same thing happens that the information coincides with the function c. The divided differences are obtained with respect to, to express in the form:$h = x_{i+1} - x_i = constante P_n f x_0 x_1 x_n f(x_i) = y_i f x_0 < x_1 < \cdots < x_n P_n(x)$

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0) \ldots (x - x_{n-1}) \ (8)$$

to find the values of. In order to determine the first of these constants,, note that if it is written in the form of the previous equation, then evaluating at leaves only the constant term, as shown in equation (9): $a_0, a_1, \ldots, a_n a_0 P_n(x) P_n x_0 a_0$

$$a_0 = P_n(x_0) = f(x_0) \ (9)$$

Similarly, when evaluating at, the only non-zero terms when evaluating obtaining the value in equation (11)$P_n x_1 P_n(x_1)$

$$f(x_0) + a_1(x_1 - x_0) = P_n(x_1) = f(x_1)(10)$$

such that

$$a_1 = \frac{f(x_1)-f(x_0)}{x_1-x_0}(11)$$

The zero-order divided difference is the function evaluation in, that is, it is simply the value of in :$f x_i f[x_0] f x_0$

$$f[x_0] = f(x_0)(12)$$

The remaining divided differences are defined inductively; The first divided difference of with respect to and is written as and is defined as:$f x_i x_{i+1} f[x_i, x_{i+1}]$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}]-f[x_i]}{x_{i+1}-x_i}(13)$$

For example,..., a part of obtaining Newton's divided differences is shown in table 2..$(x_0, y_0)(x_4, y_4)$

		a_0	a_1
x_0	$y_0 = f(x_0)$		
		$\frac{f(x_1)-f(x_0)}{x_1-x_0} = f[x_0,x_1]$	
x_1	$y_1 = f(x_1)$		$\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0} = f[x_0,x_1,x_2]$
		$\frac{f(x_2)-f(x_1)}{x_2-x_1} = f[x_1,x_2]$	
x_2	$y_2 = f(x_2)$		$\frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1} = f[x_1,x_2,x_3]$
		$\frac{f(x_3)-f(x_2)}{x_3-x_2} = f[x_2,x_3]$	
x_3	$y_3 = f(x_3)$		$\frac{f(x_3,x_4)-f(x_2,x_3)}{x_4-x_2} = f[x_2,x_3,x_4]$
		$\frac{f(x_4)-f(x_3)}{x_4-x_3} = f[x_3,x_4]$	
x_4	$y_4 = f(x_4)$		$\frac{f(x_4,x_5)-f(x_3,x_4)}{x_5-x_3} = f[x_3,x_4,x_5]$
		$\frac{f(x_5)-f(x_4)}{x_5-x_4} = f[x_4,x_5]$	
x_5	$y_5 = f(x_5)$		

Table 2. Symbolic representation of the calculation of Newton's divided differences.

Approximate error when using the method, if f is nth differentiable and continuous in and the points satisfy [a, b]

$$x_0 < x_1 \ldots < x_n \text{exists such that} \ (14)$$

$$\xi \epsilon [a,b] f[x_0, x_1 \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

Interpolation of the central finite differences using equation (4).

$$\nabla y(x_i) = y(x_{i+1}) - y(x_{i-1})\text{(fifteen)}$$

$$\nabla^2 y(x_i) = y(x_{i+1}) - 2y(x_i) + y(x_{i-1})(16)$$

$$\nabla^3 y(x_i) = y(x_{i+2}) - 2y(x_{i+1}) + 2y(x_{i-1}) - y(x_{i-1})(17)$$

$$\nabla^4 y(x_i) = y(x_{i+2}) - 4y(x_{i+1}) + 6y(x_i) - 4y(x_{i-1}) + y(x_{i-2})(18)$$

Hence the values that will be replaced directly into equation (4).$a_i, i = 1,..n$

$$a_0 = y(x_i)(19)$$

$$a_1 = \frac{y(x_{i+1})-y(x_{i-1})}{2h}(20)$$

$$a_2 = \frac{y(x_{i+1})-2y(x_i)+y(x_{i-1})}{h^2}(21)$$

$$a_3 = \frac{y(x_{i+2})-2y(x_{i+1})+2y(x_{i-1})-y(x_{i-2})}{2h^3}(22)$$

$$a_4 = \frac{y(x_{i+2})-4y(x_{i+1})+6y(x_i)-4y(x_{i-1})+y(x_{i-2})}{h^4}(23)$$

Equation (4) will be used in which the central differences are obtained using table 3. With different sinology for: y_i

Number	x_i	$y(x_i)$
y_{i-3}	310	2.4913617
y_{i-2}	320	2.5051500
y_{i-1}	330	2.51855139
y_i	340	2.5314789
y_{i+1}	350	2.544068
y_{i+2}	360	2.5563025

Table (3) symbolic representation in the first column of the y_i where they are obtained: a_i

Calculation of: a_i

$$a_0 = 2.531478(24)$$

$$a_2 = \frac{2.544068-2(2.544068)+2.51855139}{10^2} = -0.0005103322(25)$$

$$a_3 = \frac{2.5563025-2(2.544068)+2(2.51855139)-2.5051500}{2(10^3)} = -0.00638179(26)$$

$$a_4 = \frac{2.5563025-4(2.544068)+6(2.5314789)-4(2.51855139)+2.5051500}{10^4} = -2.50279651(27)$$

Evaluation of equation (4) using these values results in:

P(337.5)= 2.52638831 (28)

In the given table, the value of will be estimated using (i) Newton forward finite difference interpolation formula, (ii) Lagrange interpolation, (iii) Newton divided difference interpolation, (iv) new method using Newton's polynomial with central differences and (v). New method proposed by (Jashim Uddin eat 2019). Rounding error was used to perform the calculations.$f(x) = log_{10}\ 337.5$

Interpolation by the method ofJashim Uddin, the table shows the information used for its development.

x_i	310	320	330	340	350	360
$f(x_i)$	2.4913 617	2.5051 500	2.51855 139	2.5314 789	2.544 068	2.5563 025

Table (4) Information to perform interpolation by methods.

New interpolation method developed by (Jashim Uddin eat 2019). "Methods using forward differences Lagrange, divided differences, central differences, which are used to interpolate, for points within the set, These cannot be used to interpolate in the center of a table. to obtain more suitable results near the middle of the table, central difference interpolation methods are most preferred" Mathematically, assume that the function is the functional relationship involving the variable &. If you take the values ,,, and the corresponding values of y which are,, e, then we can obtain."$y = f(x)xyxx_0 - 2hx_0 - h\ x_0x_0 + 2hy_{-2}y_{-1}y_1y_2$

The average operatormis defined as follows:

$$\mu = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right](29)$$

average in the extreme case of central differences wheredis the Central Differences Operator to obtain the first term of the central differences, the following procedure is carried out:

$$\mu\,\delta\,y_i = \frac{1}{2}\left(E^{\frac{1}{2}}\delta\,y_i + E^{\frac{-1}{2}}\delta\,y_i\right) \quad (30\ (31)\ (32)\ (33))$$

$$= \frac{1}{2}\left(\delta\,y_{i+\frac{1}{2}} + \delta\,y_{i-\frac{1}{2}}\right)$$

$$= \frac{1}{2}[(y_{i+1} - y_i) + (y_i - y_{i-1})]$$

$$= \frac{1}{2}[y_{i+1} - y_{i-1}]$$

The third average of the central differences is given by:

$$\mu\,\delta^3\,y_i = \frac{1}{2}\left(E^{\frac{1}{2}}\delta^3\,y_i + E^{\frac{-1}{2}}\delta^3\,y_i\right) \quad (34)\ (35)\ (36)\ (37)$$

$$= \frac{1}{2}\left(\delta^3\,y_{i+\frac{1}{2}} + \delta^3\,y_{i-\frac{1}{2}}\right)$$

$$= \frac{1}{2}[(y_{1+2} - 3y_{i+1} + 3y_i - y_{i-1}) + (y_{i+1} - 3y_i + 3\,y_{i-1} - y_{i-2})]$$

$$= \frac{1}{2}(y_{1+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2})$$

The polynomial proposed by (Jashim Uddin eat 2019) is given by:

$$P_n(x) = \frac{(y_1+y_{-1})}{2} + u\left[\frac{\Delta y_{-1}+\Delta y_0}{2}\right] + \left[\frac{\Delta y_{-1}+\Delta y_0}{2}\right] + \frac{u}{2!}\left[\frac{(u+1)\Delta^2 y_{-2}+(u-1)\Delta^2 y_0}{2}\right] + \frac{u}{3!}\left[\frac{(u^2+3u+2)\Delta^3 y_{-2}+(u^2-3u+2)\Delta^3 y_{-1}}{2}\right] + \frac{u(u^2-1)}{4!}\left[\frac{(u+2)\Delta^4 y_{-3}+(u^2-2)\Delta^4 y_{-2}}{2}\right] + \frac{u^2(u-1)}{5!}\left[\frac{(u^2+5u+6)\Delta^5 y_{-3}+(u^2-5u+6)\Delta^5 y_{-1}}{2}\right] + \cdots, (38)$$

Lagrange Interpolation Method using equation (6). You have the result.

P(337.5)=2.5228284 (39)

Interpolation of Newton's Divided Differences using equation (8) calculating those in table 2.a$_i$

P(337.5)=2.5283 (40)

Method of central differences for this only the Calculation of thethat will be substituted in equation (4) evaluating at the given point.

$$a_0 = 2.531478 (41)$$

$$a_2 = \frac{2.544068-2(2.544068)+2.51855139}{10^2} =$$

$$-0.0005103322 (42)$$

$$a_3 = \frac{2.5563025-2(2.544068)+2(2.51855139)-2.5051500}{2(10^3)}$$

$$= -0.00638179 (43)$$

$$a_4 = \frac{2.5563025-4(2.544068)+6(2.5314789)-4(2.51855139)+2.5051500}{10^4}$$

$$= -2.50279651 (44)$$

P(337.5)=2.52638831 (45)

ForJashim Uddin uses the following procedure:

Here = 10, since we will find y = log10337.5. Let's take 330 as the origin. = 0.75hu

$u = \frac{x-x_0}{h}$, where,

$$x = 337.5, x_0 = 330u = \frac{337.5-360}{10} = 0.74$$

x	u	y	Δy	Δ^2y	Δ^3y	Δ^4y
310	- 2	2.4913617				
			0.0137883			
				-0.0004244		
320	-1	2.5051500				
			0.0133639			
					0.0000255	
330	0	2.5185139		-0.0003989		-0.0000025
			0.012965			
			0.0125891		0.000023	
340	1	2.5314789		-0.0003759		
			0.0122345			-0.0000017
					0.0000213	
350	2	2.544068		-0.0003546		
360	3	2.5563025				

Table (5) Shows the calculation of the differences to use the proposed method

Using equation (38).

$$P(337.5) = \frac{2.5314798+0.50515}{2} + \frac{.75}{2}(.0133639 + .012965) + \frac{.0133639-.0122965}{2} + \frac{0.75}{2}\left[\frac{2.5314789+.50515+(.75+1)x(-.0004244)+(.75-1)x(-.00003759}{2}\right] + \left[\frac{.75^2+3x.75+2)x.0000255+\left((.27^2-3x0.75+2)x(-.0003759)\right)}{2}\right]x\frac{0.75}{6} \quad (45)$$

Performing the arithmetic operations we have

P(337.5) = 2.52827357 (46)

P (337.5) = 2.51831445+ 0.0098733375+

0.00019945- 0.0001216359375-
0.00000012109375 (47)
P(337.5) = 2.52827358(48)

The following table (5) presents the values obtained by the methods.

Method	True value	Approximate value	Percentage Error
Interpolation of Newton's finite differences	2. 528273777	2.528278	0.00016315893
Lagrangian interpolation	2.528273777	2.5228243	0.215541442207
Interpolation of Newton's Divided Differences	2.528273777	2.5283	0.00103718989
Method using central finite differences	2.528273777	2.52638831	0.0000188547
New method proposed by (Jashim Uddin eat 2019)	2.528273777	2.52827358	0.00000779187

Table (5). Percentage error committed by the methods.

RESULTS AND ANALYSIS

From the results of table (5). It is observed that the proposed method of central finite differences shows a smaller error than using the new method of Jashim Uddin, in which more error is committed is the Lagrange method. The other alternatives would be to perform the calculation by the methods of finite differences and Newton's divided differences, but these have a problem when the calculation is carried out by hand, a mistake will produce a larger error due to the rapid propagation of the error. but the best is the proposed new method of central differences. Jashim Uddin comes second.

CONCLUSIONS

The methods finite differences, divided differences of Newton and Lagrange were programmed in C language, Lagrange interpolation due to the number of operations error propagation increases rapidly when the degree of the polynomial increases depending on the number of points, for example, if there are 26 points. The method becomes very unstable. On the other hand, the other methods have a better approximation of the interpolation value, but the one that made the least error was the proposed method of central finite differences, but all of these must use all the information, the one proposed by(Jashim Uddin eat 2019) needs fewer points, but makes a greater error than the previous one, but it is an alternative so that when you have points the others perform more arithmetic operations.

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