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CONSTRUCTION OF AN RL CIRCUIT, MODELING THE GOVERNING DIFFERENTIAL EQUATION AND COMPARISON BETWEEN THE ANALYTICAL SOLUTION AND EXPERIMENTAL DATA

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Abstract: This article proposes the modeling of the ordinary differential equation referring to a resistor-inductor (RL) electrical circuit, obtaining the analytical solution and comparison with experimental data (charge and electrical current) obtained from an electrical circuit that It was built with recyclable materials. This practical approach aims to sharpen the student's interest in the discipline and the undergraduate course, offering them another incentive for your stay at the university. As in other disciplines, teaching the theory of ordinary differential equations (ODE) can be associated with practical experiments, allowing the connection between theory and practice, providing opportunities for efficient learning of the content covered and the perception of how mathematics is embedded in various events around us. Furthermore, mathematical modeling is an interesting, powerful and enlightening artifice, as it allows a better visualization of the laws and properties that govern the studied event. Following this path, this work contemplates the modeling of the ordinary differential equation that governs the movement of charge and current in an RL electrical circuit, its analytical resolution and the comparison between the analytical results and experimental data obtained from the electrical circuit. This way, Kirchoff's second law and concepts from the theory of electricity were used to model the ordinary differential equation that governs the RL circuit. The analytical solution of this problem was obtained by the integrating factor method, as the ordinary differential equation governing the electrical circuit has the characteristics of being linear and first order. The construction of a circuit was carried out using recyclable materials that can be found in homes, schools, universities or in places that receive electronic waste. The construction of the circuit made it possible to obtain experimental data (charge and electric current), which were measured using a multimeter. Subsequently, these data were compared with those obtained by the analytical solution. The relative error obtained shows the compatibility between the data obtained by the experiment and the analytical solution. The objective of this work is to bridge the gap between theory and practice. This way, the text has a practical educational teaching and learning nature, bringing to the public the importance of associating concepts and properties with problem solving.
Keywords: Mathematical modeling. Ordinary Differential Equation. RL Circuit. Teaching and learning.

INTRODUCTION

In the first semesters of undergraduate courses in Mathematics, Chemistry, Engineering, Physics and others, students experience difficulties in the calculus subject. These difficulties add up later in the discipline of differential equations. In order to alleviate these problems, awaken the student to new teaching approaches and encourage research, a closer relationship between the theoretical concept and its application is sought, allowing students, in addition to understanding the content, practical visualization in their area of study. degree (CARGNELUTTI; GALINA, 2015).

According to Oliveira and Igliori (2013), the subject differential equations brings many challenges to students, mainly due to the large amount of content seen in other subjects that are necessary for their good understanding. Furthermore, the student needs to understand the intrinsic concepts and techniques of the discipline and this process may present less difficulty when combined with a contextualized practical application.

It seems appropriate that the school can promote teaching in which students have the possibility of understanding the laws and properties related to the content, knowing and relating their representations and using them to interpret facts of reality. With this perspective, mathematical modeling presents itself as an efficient pedagogical alternative (VERTUAN, 2007).

According to Schmidt, Ribas and Carvalho (1998), teaching must be more than just the transmission of content. Expand and enhance the development of skills so that the student can reflect on the issues that surround them and how mathematics is inserted in many events.

JUSTIFICATION

In this sense, this text presents the modeling and analytical resolution of the differential equation that governs the movement of charge and current in an electrical circuit (RL). Furthermore, the analytical solution is compared with data from measurements on the electrical circuit that was built with recyclable materials.

THEORETICAL REFERENCE

DIFFERENTIAL EQUATIONS

According to Boyce and DiPrima (2006), a differential equation is a law, or a prescription, that establishes the rate at which things happen. Expressed in mathematical language, rates are derivatives and relationships are equations that govern the behavior of some physical process.

According to Neto (2021), ordinary differential equations were initially introduced by Isaac Newton (1646-1727), in the century, through studies carried out on differential and integral calculus, and later by Gottfried Wilhelm (1646-1717), Leibniz, in the 17th century. Leibniz made important contributions to the study of ordinary differential equations, established the method of separating variables, reducing homogeneous equations, and the procedure necessary to solve first-order linear equations. First order linear equations and their applications are the first topics to be studied in the ODE discipline, consequently they have numerous applications, in this case we will use one of the methods, known as integrating factor, to solve problems related to RL circuits.

For Konzen (2023) "Differential Equation (DE) is the name given to any equation that has at least one term involving the differentiation (derivation) of an unknown" (Ibid, p. 1).In other words, it is an equation that involves derivatives of one or more dependent variables related to independent variables.

When Calculus problems are introduced, it often causes strangeness among students, as it is difficult to visualize content. When this occurs, one of the ways to improve this visualization is to use interactive, or expository, activities to stimulate the student's interest and understanding of the content taught. For Moreno and Azcárate (1997; 2003), what determines the way the teacher acts in the classroom can be based on three different teaching models. The first is one that is based on traditional teaching, that is, the application of analytical techniques and solving differential equations. The Second is more improved, as it considers differential equations an instrument that aims to mathematically model practical problems, and the resolution of these problems takes place through graphic, numerical and symbolic representations. The last model has a transitory characteristic, that is, the teacher conflicts about "what he does" and "what he could do".

The teaching of Differential Equations follows the traditional teaching model, but when we talk about modern times it is notable that higher education has undergone some changes. According to Dullius, et al. (2011), the context in which differential equations are introduced today is different from what it was half a century ago, as now the demands and needs of students are different, in addition to technological advances. Despite this, the context in which differential equations are presented remains as it was half a century ago, not keeping up with the changes that accompany the phases of society, classes must be rethought, in addition, technological advances must be considered.

MATHEMATICAL MODELING

Mathematical Modeling "[...] consists of the art of transforming reality problems into mathematical problems and solving them by interpreting their solutions in the language of the real world" (BASSANEZI, 2011, P. 16).

This way, we can interpret mathematical modeling as a process in which everyday life situations are mathematically articulated, that is, they are translated into mathematical language. Modeling is fundamental in solving practical problems since this will lead to the resolution of real problems. Biembengut and Hein (2014), highlight that mathematical modeling "[...] It is an art, when formulating, solving and elaborating expressions that are valid not only for a particular solution, but that also serve, later, as support for other applications and theories" (Ibid, p. 9).Therefore, mathematical modeling can be used, not only to obtain a certain solution, but also to incorporate new knowledge or to acquire new skills based on the knowledge you have.

Following this line, the modeling of the ordinary differential equation that models an RL electrical circuit is presented below.

RESISTOR-INDUCTOR CIRCUIT

An electrical circuit is a closed path in which the electrical elements of the circuit are connected by a conductive medium. An electric current pass through these components causing a potential difference in each component (IRWIN; NELMS, 2013).

Electric current is defined as the flow of electrically charged particles that move from one pole of one component to another.

Potential difference, also known as electrical voltage, is the difference in electrical potential between two points in a circuit. Its unit of measurement is Volts and can represent both an energy source and "lost" or stored energy (voltage drop).

The elements that make up the circuit determine its classification. In this work, only RL type circuits will be used. These are made up of resistors and inductors.

For the study of Applications of Equations Differences in ordinary RL electrical circuits it is necessary to recall some basic concepts of electricity.

Ohm's First Law states that if a conductor with constant electrical resistance is kept at a constant temperature, the electrical current intensity will be proportional to the potential difference applied between its ends:

$$R = \frac{V}{i} \quad (1)$$

where, R is resistance, measured in ohms (Ω), Vis the voltage/charge, measured in volts and i is the current, measured in amps.

Kirchhoff's two laws are indispensable for studies of the RL circuit, as follows: The First law establishes that the sum of the current intensities that arrive at a node is equal to the sum of the current intensities that leave the node. The second law considers that the algebraic sum of the voltages found in each element of the circuit is zero.

The expression to obtain the Inductance L is given by:

$$L = \frac{\mu_0 N^2 A}{l} \quad (2)$$

where, L is the inductance of the coil, measured in Henry, N is the number of turns in the coil, A is the area of the coil core, measured in square meters [m2], l is the length of the coil, measured in meters [m] and μ_0 It is the magnetic permeability of air which is equivalent to $4\pi 10^{-7}$[H/m].

Electric current expressed through derivative is also necessary. It is defined as the intensity of the electric current i the rate of change of electrical charge q in relation to time t that passes through a cross section of a conductor, being:

$$i(t) = \frac{dq}{dt} \tag{3}$$

According to the recyclable materials obtained, the RL electrical circuit was built as shown in Figure 1.

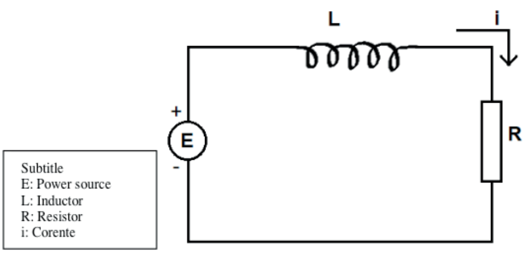

Figure 1: RL Circuit.

Source: From the authors (2023).

Kirchhoff's current law implies that the same current passes through each element of the electrical circuit. Applying Kirchhof's voltage law to this circuit, which establishes for an RL circuit that the sum of the voltage drops across the resistor R and the inductor L must be equal to the supplied voltage E:

$$L\frac{di}{dt} + Ri = E \tag{4}$$

Dividing equation (4) by L what:

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \tag{5}$$

Expression (5) is an inhomogeneous linear first-order ordinary differential equation and can be solved by the integrating factor method. This method is used to solve first-order linear ODEs.

ANALYTICAL RESOLUTION

The ordinary differential equation (5) has an analytical solution and is then obtained using the integrating factor method (ZILL, 2006).

To apply this method, the ODE must be in the form:

$$\frac{dy}{dt} + p(t)y = f(t) \tag{6}$$

Observing equation (5), we have that:

$$p(t) = \frac{R}{L} \tag{7}$$

It follows, therefore, that the integrating factor has the following form:

$$\mu(t) = e^{\int \frac{R}{L}dt} \tag{8}$$

Solving the integral of (8), we have:

$$\mu(t) = e^{\frac{R}{L}t} \tag{9}$$

Multiplying (9) into (5), it follows that:

$$e^{\frac{R}{L}t}\frac{di}{dt} + e^{\frac{R}{L}t}\frac{R}{L}i = e^{\frac{R}{L}t}\frac{E}{L} \tag{10}$$

Multiplying the integrating factor in (10) made the left side of the product rule between the integrating factor and the current. Therefore, expression (10) can be rewritten as follows:

$$\left(e^{\frac{R}{L}t}i(t)\right)' = e^{\frac{R}{L}t}\frac{E}{L} \tag{11}$$

In this step, we proceed with the integration of both sides of expression (11). On the left side, integration is direct (property of integrals). On the right side, integration by substitution applies. There is the expression

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for the electric current that circulates in the circuit:

$$e^{\frac{R}{L}t} i(t) = \frac{E}{R} e^{\frac{R}{L}t} + k \tag{12}$$

where k is the integration constant.

The objective of solving (5) is to obtain the current flowing through the circuit. Thus, isolating the current in (12), we have:

$$i(t) = \frac{\frac{E}{R} e^{\frac{R}{L}t} + k}{e^{\frac{R}{L}t}} \tag{13}$$

Expression (13) can be rewritten:

$$i(t) = \frac{E}{R} + k e^{-\frac{R}{L}t} \tag{14}$$

The constant integration of (14) can be determined considering that in time t=0 There is no current circulating in the system. So: i(0)=0. It is $k = -\frac{E}{R}$.

Therefore, expression (14) becomes:

$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{15}$$

By definition, the rate of change of load q in relation to time t is equal to current, as shown in (3). Follow that:

$$\frac{dq}{dt} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{16}$$

Expression (16) is a separable ODE and can be rewritten:

$$dq = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) dt \tag{17}$$

Expression (17) can be solved by the variable separation method. Applying indefinite integration on both sides, we have:

$$\int dq = \int \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) dt \tag{18}$$

Therefore, the expression for the electrical charge in the RL circuit is given by:

$$q(t) = \int \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) dt \tag{19}$$

MATERIALS AND METHODS

To build the RL electrical circuit, a computer source was used as a power source, as shown in Figure 2.

Regarding the resistor, we chose to use a computer cooler/fan, as shown in Figure 3, as it is an object easily found in electronics waste and also because of its practicality in handling. A resistor is an electrical component that has the primary function of limiting the flow of electrical current in a circuit.

The inductor was constructed from a 0.5-millimeter-thick copper wire, winding this wire on a rod approximately one centimeter thick. Thus, we have a coil, as can be seen in Figure 4. The inductor, in addition to the coil, has a core, which can be composed of a material such as metal or an insulating material. It was decided to use air as an insulator.

With all the components in hand, plus some other components such as wires and insulating tape, the electrical circuit diagram shown in Figure 1 is used to build the circuit used in this work. This way, we have the circuit shown in Figure 5.

Current and electrical charge measurements were made using a multimeter.

The multimeter used is shown in Figure 6.

With this device you can measure current, voltage/charge and resistance. To do this, simply select the correct option on the center spinner button. To measure current and voltage, Kirchhoff's law was used. Therefore, to measure voltage it is necessary that the

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Figure 2: Source used in the experiment.

Source: From the authors (2023).

Figure 3: Cooler/fan used as resistor.

Source: From the authors (2023).

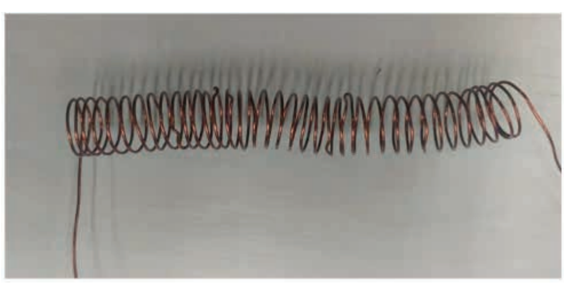

Figure 4: Built inductor.

Source: From the authors (2023).

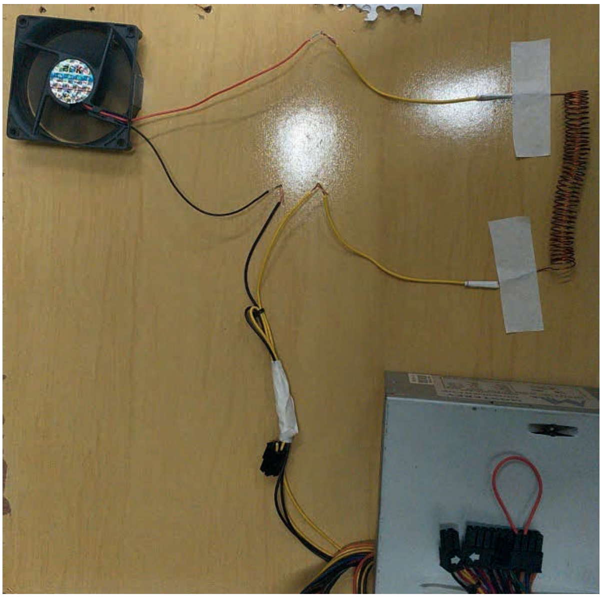

Figure 5: RL circuit built and in operation

Source: From the authors (2023).

Figure 6: Multimeter used.

Source: From the authors (2023).

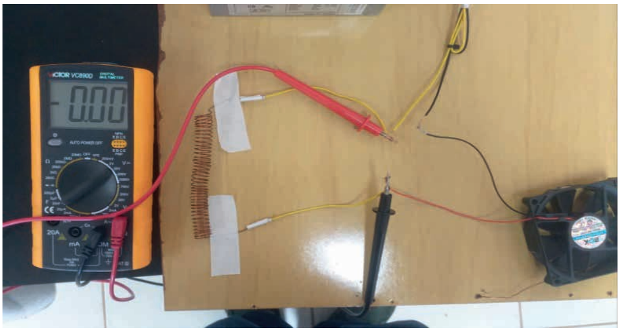

Figure 7: Inductor voltage

Source: From the authors (2023).

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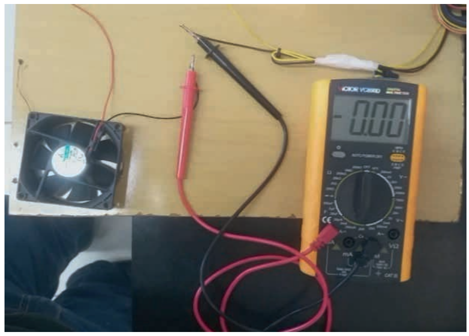

Figure 8: Current in the circuit

Source: From the authors (2023).

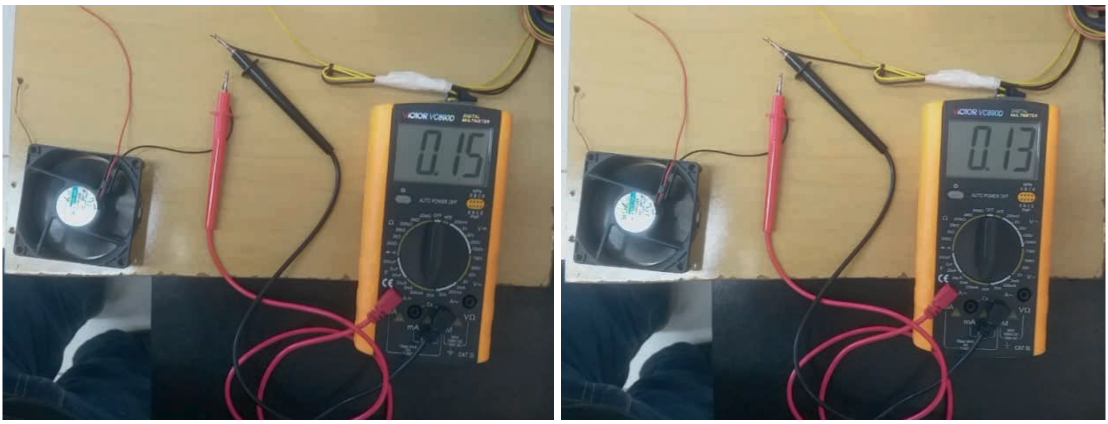

Figure 9: Current measurement as soon as the circuit has been turned on.

Source: From the authors (2023). Source: From the authors (2023).

multimeter is in the appropriate configuration. To measure the current, with the multimeter in the correct configuration, it is necessary to leave it in series with the circuit, as shown in Figures 7 and 8.

RESULTS AND DISCUSSIONS

For the proposed problem, the data from the components of the constructed RL circuit measured with the multimeter are: The computer source has a load of 12 V (volts) and the fan indicates a consumption of 0.16 A. Substituting these values in formula (1), we have:

$$R = \frac{12}{0{,}16} = 75\Omega \tag{20}$$

With the construction of the inductor, it was necessary to calculate the inductance. To do this, we substituted the values corresponding to our inductor in expression (2), obtaining:

$$L = \frac{4\pi 10^{-7} 39^2 \frac{9\pi}{250000}}{\frac{29}{250}} = \frac{13689\pi^2}{72500000000} H \tag{21}$$

The calculation of the current in the RL circuit at a certain time t according to Kirchhoff's voltage law is provided by expression (5). Substituting the data from (20) and (21) into (5), we have that:

$$\frac{13689\pi^2}{72500000000}\frac{di}{dt} + 75i = 12 \tag{22}$$

Expression (22) is rewritten to apply the integrating factor method, obtaining:

$$\frac{di}{dt} + \frac{5437500000000}{13689\pi^2} i = \frac{870000000000}{13689\pi^2} \tag{23}$$

The form of the solution to (23) was presented in equation (15). This way, the value for the current is obtained, analytically:

$$i(t) = \frac{4}{25}\left(1 - e^{\frac{-5437500000000t}{13689\pi^2}}\right) \approx \frac{4}{25} = 0{,}16\,A \tag{24}$$

Once the analytical part of the proposed problem is complete, we proceed with measuring the electrical current in the circuit using the multimeter. These measurements can be seen in Figures 9 and 10.

The relative error is given by dividing the absolute error and the analytical value. It is observed, in the measurements, that as soon as the circuit was connected, the current was 0,15 A. One minute later the current oscillates between 0,13 A It is 0,14A. The relative errors for the measurements as soon as the circuit was turned on and one minute later are presented below:

$$E_{rel} = \frac{0{,}16 - 0{,}15}{0{,}16} = 0{,}0625$$

$$E_{rel} = \frac{0{,}16 - 0{,}135}{0{,}16} = 0{,}15625$$

CONCLUSION

It is observed that the relative error calculated immediately after turning on the circuit is smaller than the relative error one minute later. This difference may be due to several factors, for example, the multimeter consumes a little energy from the circuit, and there is also wear on the copper wire, which was disregarded during the calculations. Furthermore, small errors may occur when measuring due to the precision of the device and also due to the fact that everything was done manually.

The objectives were achieved, as the analytical solution, which was obtained through solving the first-order linear ODE, and the experimental data are close, as shown by the relative error, showing that the theory can be observed in events around us. Furthermore, the learning experience was great. Observing, in practice, mathematical concepts and properties, made the learning process more efficient, arousing interest and providing motivation in students. Interdisciplinarity in the development of this work is also a relevant item.

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