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TRIANGULAR INEQUALITY A FORGOTTEN PROPERTY BY MATH TEACHERS

Luis Leonardo Argueta-Mogollón
PhD in Innovation and Educational Technology
Faculty of Humanities, ``Universidad de San Carlos de Guatemala``

Abstract: The use and applicability of the triangle inequality theorem and the Pythagorean theorem to construct and identify triangles in mathematics class was investigated. The qualitative research and descriptive scope yields valuable information regarding teacher training on the construction of triangles and their mastery of the subject. It was required to use concrete material for problem situations in the construction of the mathematical object. Five mathematics teachers from a private urban educational center in Sololá, Guatemala, were selected in a non-probabilistic, incidental and convenience manner because they have different university backgrounds and have more than four years of teaching experience.
Keywords: Triangle, triangular inequality, Pythagorean Theorem.

INTRODUCTION

In Guatemala, the national base curriculum, CNB, establishes the learning of the triangle, the teachers carry out different activities for the learning of different geometric figures, including this mathematical object. Learning meaningfully about the triangle is not an easy task, and it starts with a property called triangle inequality; It is not just learning that a triangle is built with three segments or lines; it is necessary that they meet certain requirements and that are indicated in the triangle inequality theorem.

The professors in diversified work the area of trigonometry, the Pythagorean theorem for right triangles is highlighted, but they forget its extended application to other triangles. There is no classroom work on triangular inequality and therefore there are cognitive gaps to build a triangle and identify its properties, even before drawing or building it. This lack of knowledge affects the educational process in advanced trigonometry courses, so it is important to highlight this problem, the established objective of this study.

The research was carried out with teachers from a private educational center in the urban area of Sololá because students from different municipalities of that department come together there, it is a recognized school in the community and in the tests that the Ministry of Education of Guatemala carries out year after year, it has obtained acceptable results. The professors have had university studies in administration, architecture and professorships in mathematics and computing, in addition they have more than 4 years teaching this subject.

A bibliographical consultation was carried out about the construction of triangles, triangular inequality and the expanded use of the Pythagorean theorem to recognize the types of triangles by their angles; the textbooks and the themes established by the CNB for this geometric figure were reviewed; The teachers of the educational center discussed the importance of identifying the type of triangle. It is important to highlight the need to learn triangular inequality, a subject that is not specified in the CNB, nor in textbooks, but necessary to master the construction of triangles; establish the level of mastery of this property by them and propose strategies that allow significant learning about this topic.

This qualitative study, with a descriptive scope and based on the question: What are the problems that high school mathematics teachers show in relation to triangular inequality? It was carried out from February to May of the year 2022 with the participation of five mathematics teachers of that educational level and informal talks were held about the importance of identifying the types of triangles in university studies; The main limitation of the study was time due to the workload they have, which is why the director established maximum days and time for contact with teachers within the educational center.

In the Sololateco context, especially in secondary school, no studies have been carried out on triangular inequality; For this reason, this research is novel and shows shortcomings in the teaching of mathematics. It is important to emphasize that the "superficiality in the teaching of trigonometry in high school fosters a series [sic] incidence in the learning of calculus at the university level" (Aray Andrade et al., 2020, p. 66) and is a cause of student failure in higher mathematics courses; prototypical figures have great influence; for this reason, Clemente et al. (2017) indicate that "the prototypical image of geometric figures that students have generated throughout their school experience influences their ability to recognize them or to build certain geometric objects during problem solving" (p. 499).

To overcome these failures in the educational process, it is necessary for the teacher to become aware that "the traditional teaching of Trigonometry, the figures drawn by the teacher on the blackboard, in addition to being static and rigid, can be very different from what he wants to represent" (González Posada Acosta et al., 2017, p. 403), make the didactic changes that are required, master the theme and carry out the connections mentioned by Gueudet and Quéré (2018), through exercises and problems that they not only repeat algorithms and techniques seen in class, but also contribute to the significant learning of the triangle and its construction; in many cases they will require putting the textbook aside.

For Krajcevski & Sears (2019) there is a need to abandon the prototypical figures a bit and present the student "in their learning environment different types of triangles (acute, obtuse right, isosceles...), placed differently within the coordinate system " (p. 99); Some teachers will try to introduce technology, but as Arévalo Duarte (2016) mentions, quality teacher training is always essential, because they favor the successful implementation of ICT in learning processes and not just mechanically; For this reason, the combined use of technology with concrete material is insisted on.

It is necessary to identify the failures in the educational process of the triangle; "Improving the quality of Mathematics teaching and learning, with special emphasis on working with the Pythagorean theorem" (Conde-Carmona and Fontalvo-Meléndez, 2019, p. 259) is urgently required and now in the post-pandemic stage "The teacher must also master technology (T), be able to provide technological support to students and other human resources" (Sampaio, 2016, p. 215). It is time to recover, indicate Montiel and Jácome (2014), that geometric process in the construction of the trigonometric; in this case, something as simple and forgotten as the triangular inequality.

Something that also makes it difficult to master the properties of the triangle are the simple exercises that are solved in the mathematics classroom. Rahaju et al (2019) mention that future primary school teachers have difficulties recognizing triangles in problem situations other than the prototypical ones and indicate the existence of teachers who cannot solve triangle problems; They indicate that one of the causes is learning restricted to the figures in textbooks, which for the most part do not require analysis, but only a mechanical process of solution; Finally, Soto-Ardila et al. (2018) that in learning the triangle the management of the intrinsic properties is needed and one of them is the triangular inequality.

TRIANGLE, THE SIMPLEST POLYGON

Learning about trigonometry in the diversified cycle of the middle level requires making use of several previous knowledge

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and the simplest of them is the construction of a triangle, identifying its properties, be they by the measure of its sides or by the measure of its angles must be an easy task, but mechanical. Their learning is established in the basic national curriculum from primary school; Teachers consider that a student at the beginning of high school, after more than six years of working in the classroom, anyone is capable of identifying him by his shape, characteristics and properties.

The triangles from their construction by the measurements of their sides can be classified as scalene, that is, the three sides have different measurements. By containing two equal sides and two angles opposite them, which have the same value in degrees, it is called an isosceles triangle. Finally, the equilateral triangle is defined as a polygon with three equal sides and also three equal angles; These definitions are based on the document written, in 2006, by Professor Diana Barredo Blanco, who prepared it for proper handling of this mathematical object in the classroom.

Triangles also have a name according to the type of angles it contains; the acute angle whose angles are all acute; that is, less than 90 degrees. An example is the equilateral triangle where all its angles measure 60º.

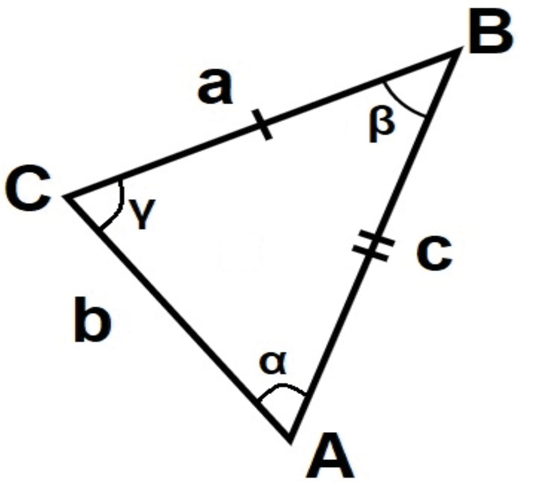

Figure 1 *Triangle in a different position than the prototype.*

An object of variety of studies is the right triangle, has in its structure an angle of 90º and the denomination is completed with the obtuse triangle, it is one that necessarily has an obtuse angle, that is, greater than ninety degrees and therefore mnemonically this classification is called ARO for acute angle, rectangle and obtuse; everything in relation to the minor angles, equal to or greater than 90º; ARO makes it easier to internalize that in the acute triangle all three angles are acute; the right triangle contains two acute angles and one right angle, equal to 90º; now, the obtuse angle integrates two acute angles and one greater than 90º.

Figure 1 shows a triangle in a different position from the prototypical one, a polygon, the one with the fewest sides, according to Barredo Blanco, (2006), is defined as a polygon with three sides and three vertices. At each vertex two sides of the triangle meet and are denoted by capital letters. Side a is the segment joining vertices B and C; that terminology and nomenclature must be handled properly by the student who finishes the primary level. Something trivial, but easy to forget, is that the length of any of the sides of that figure will always be less than the sum of the other two; this is called the triangle inequality property.

THE RIGHT TRIANGLE AND ITS TRADITIONAL REPRESENTATION

The right triangle is a mathematical object extensively studied in human history; according to Alvarez et al. (2015) the world population, regardless of the place, will be able to understand those written in mathematical language, because it is universal; they present a brief history of its evolution and mention some examples, including the Rhind Papyrus, 17th century BC. "A triangle with 10 jets at its edge and 4 jets at its base, what is its area?" (Álvarez et al., 2015, p. 11) the use of the word base encourages the student to think of a vertical triangle, a prototypical position in textbooks.

The memorized concepts and the use monopolized by the prototypical figures have caused failures in learning; the internet has caused a greater overexposure to these figures; in figure 2 right triangles are shown in their prototypical form; all of them taken from the Google browser in a simple search for that mathematical object; It is important to keep in mind that geometry is an important part of the educational process of the human being; In his first years of school, the child plays with objects that allow him to learn about it. They must learn significantly about this area of mathematics and as Camargo Uribe (2011) mentions, it is important to "pay attention to the identification of a series of properties of figures, such as the existence of corners and curves, the simplicity and familiarity of students with them" (p. 47).

It is necessary that the teacher when teaching include the construction of triangles because the apprentice will face problems related to them in his daily life; mentioned Rodríguez Palmero et al. (2008) that "the long-term acquisition and retention of organized bodies of knowledge" (p. 9) is important for life; but in the triangle there is a disintegration of the appearance of the figure, the concepts and the mathematical vocabulary; As an example, the term corner used by teachers of the first years of schooling is maintained for many years and up to several years is associated with the term vertex. Vocabulary used in pre-school is maintained even in high school.

EVERYDAY LANGUAGE AND MEMORIZATION OF MATHEMATICAL CONCEPTS

Students from elementary school define the triangle as a figure with three sides; It is not associated with the word polygon, which etymologically means many angles, from the Greek polys which indicates a lot and gonos which translates as angle; much less the rest of properties and terminology. Teachers use prototype figures and on many occasions do not use the object in other positions, rotation and/or reflection; much less require the student a point of reference that allows an adequate relationship between the vocabulary with the problem situation; All this causes failures to identify geometric figures based on their properties, even with a triangle, the simplest polygon.

To overcome this situation, it is necessary to return to "useful knowledge, at first: first count, then measure, then calculate -which is nothing more than knowing without counting or measuring-" (Álvarez et al., 2015, p. 7) in relation to with the geometric figures, their parts and some definitions that are considered the basis for an adequate mathematical vocabulary, similar to the term corner and vertex indicated above. Relating everyday language, terminology and its applications within the classroom requires the teacher to plan, organize and generate didactic situations on the same mathematical object.

In the case of the triangle, not only geometric figures already built must be presented, in different positions, rotations and reflections, but also problems of its construction to allow new opportunities, experiences, discoveries and solidification of the acquired knowledge. The teacher must ensure that the triangular inequality property has been internalized by the learners; remember that "One of the parts of mathematics that gives more words to everyday language, perhaps because it is usually the one that the ordinary man dominates or remembers the most about his relationship with it, is Geometry" (Muñoz Santonja, 2010, p. 91) which causes "situations in which the number of mathematical words, used correctly or not, are overwhelming" (Muñoz Santonja, 2010, p. 92) and this must be taken into account by them at any educational level.

When a mathematics teacher is aware of this wealth of vocabulary; He will provide situations in the classroom so that students, in addition to the abstract, learn geometry, trigonometry and their applications in everyday life. This implies that: in addition to identifying geometric figures, manipulating them with concrete material in the classroom, appropriate instruments are also applied to build, draw and/or measure them; The work must be accompanied by vocabulary according to the given problem or situation. Teachers will plan activities that allow for that vocabulary and knowledge in relation to triangular inequality.

There is plenty of studies on the triangle, especially the right triangle; in academic Google with an advanced search with the exact phrase "Rectangle triangle" 11,100 results are found, but if it is about triangular inequality in the last five years it is reduced to 1010 documents; Figure 3 presents screenshots of the search carried out and highlights the great difference in scientific articles in relation to both topics; but especially the little handling of the theorem of triangular inequality.

In the classroom it must be clear that geometry "etymologically, is the measurement of the earth. As a branch of mathematics, it studies the extension, shape and position of the figures, in broad strokes" (Melchor Aguilar, 2013, p. 14) and Castellanos (2014), on geometry, adds "studies the intrinsic properties of the figures (those that do not change with their movement)" (p. 1); The same figure can be in a different position, but having mastery of its properties makes it relatively easy to identify it.

Trigonometry must be understood, according to Melchor Aguilar (2013), as the measurement of triangles; others mention that it is to measure three angles and this is related to that figure; For Caro and Zamudio (2011) the angle is the union of two rays called sides and that have an extreme point in common, called the vertex and that the common man calls a corner, especially if it is a right angle.

It is important that the student body learn significantly what a triangle is, that they can identify it based on its equilateral, isosceles or scalene sides; mnemonically EQUISES; or recognize it based on its angles, mnemonically mentioned as ARO; they will be more efficient in solving problems related to this mathematical object, but they must also properly handle triangular inequality; otherwise they will face situations that they will not be able to solve; The teacher on his part must create the spaces so that this property is mastered by each and every one of those who participate in their educational encounters within geometry and trigonometry.

TRIANGLE INEQUALITY, AN ELUSIVE PROPERTY IN THE CLASSROOM

The construction of the triangle requires keeping in mind the triangular inequality, that property that indicates that the measure of any side will always be less than the sum of the lengths of the other two sides. González Polo (2017) mentions that within the curriculum the learning of the triangle is specified from the second grade of primary school, the shape, space and measurement are studied, its characteristics are also described by the shape of its sides, but its construction is not mentioned. The use of technology affects the construction and acquisition of this property; Borrego et al (2019) mention that when working on triangular inequality with technology, specifically the Geogebra software, students are able to build triangles, but what is implicit in the software regarding this property is not reflected; add that, when presenting an activity, in relation to it, for the

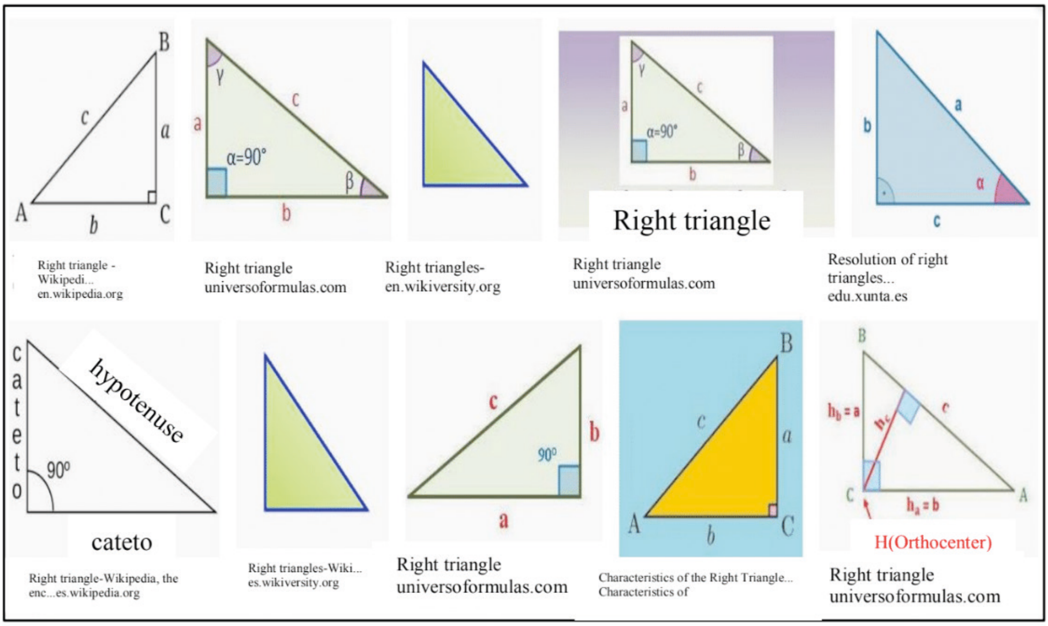

Figure 2. *Rectangular triangles in their prototypical form.*

Source: Google, search with the Right Triangle parameter.

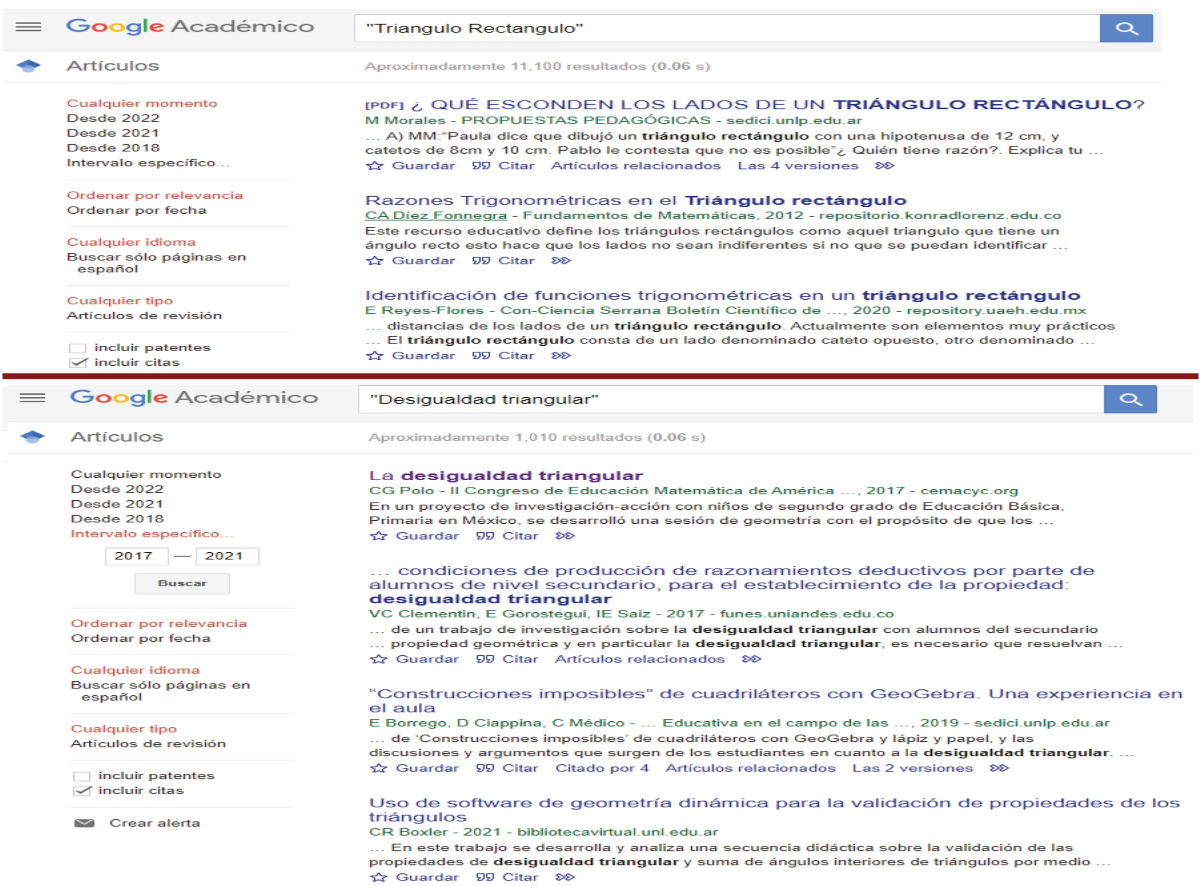

Figure 3 *Screenshots of searching for the term triangle in Google Scholar*

construction of a parallelogram, the students did not notice the approximations made by the software.

In GeoGebra: make the parallelogram using properties and the measures of the corresponding sides, then draw the diagonal and 'squash' it with the 'Pick and Move' tool until the diagonal measures 10. This is possible to visualize in the software because to the decimal approximation predetermined in it. (Borrego et al., 2019, p. 7)

Then it is evident that the use of prototypical figures, already built in the textbooks within the classroom activities, together with those hidden supports that the computer programs present cause gaps in concepts, definitions and properties; Fernández Molinero and Río Cabeza (2015), from ``Universidad de Granada``, indicate that "An adequate manipulative material will help to avoid and correct errors, since the students themselves will be the ones who carry out their constructions" (p. 387); that is to say, it is always necessary to prepare activities with concrete, manipulative material, to reduce those shortcomings in students when building knowledge.

It is recommended, based on the previous quotes, that teachers do not settle for working on the exercises in the textbook, that they also opt for technology and consider that there are no shortcomings with it; It is imperative that teachers return to the use of concrete materials and tools of yesteryear to consolidate the concept of triangular inequality. In many cases, the use of concrete material is avoided with the excuse of how expensive they are, however "The use of these materials allows the manipulation of geometric objects, an intuitive approach to the geometry of the plane and the construction processes involved are logical and efficient" (Fernández Molinero and Río Cabeza, 2015, p. 390).

The teacher, of course, must take his time to select material that is in the environment, that is easy to acquire, in order to make it available to the student body at any time; is, as much as possible, low cost and the student can use it in her own home. Once the instruction, review exercises and problem situations that the student is asked to do at home to consolidate knowledge, have been completed, then they will master triangular inequality.

METHODOLOGY

The general question posed at the beginning of the study was: What are the problems that high school mathematics teachers show in relation to triangular inequality? It is a qualitative research, at a descriptive level; Five teachers participated who, in the year 2022, taught mathematics in the basic cycle as well as in the diversified one; both daily and weekend at the School of Computer Science. For accessibility, the research was carried out in this educational center, the selection was non-probabilistic and for convenience.

Teachers were presented with a questionnaire identified as Instrument prf-05 that contained 6 problem situations; in each of them 3 segments were presented. The problem situation TE01 allows the construction of an equilateral and acute triangle by definition, TE02 a scalene right triangle, TE05 presents the difficulty of not having traditional measurements or of prototypical triangles and it is scalene on its sides, but obtuse-angled by angles; while in the last one, TE06, an obtuse-angled isosceles triangle is presented.

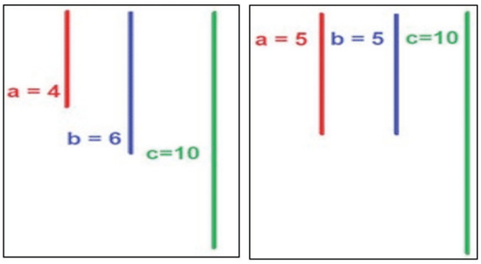

Figure 4 Problem situations TE03 and TE04

Problem situations TE03 and TE04 require mastery of triangular inequality; because you can not build a triangle; the first of them presents three segments with measures 4, 6 and 10; shows the participant a possible resemblance to the right triangle and prompts to identify those segments for a scalene triangle; while in situation TE04 he is presented with segments 5, 5 and 10 to verify if he associates it with an isosceles triangle.

In each problem situation, you are asked to answer the questions: Do you build an equilateral, isosceles, or scalene triangle with these three segments? Do you build an obtuse, right, or acute triangle with these three segments? And in each of them you are asked to justify your answer. Finally, the measurement of the perimeter is asked with the question What is the measurement of its perimeter? To corroborate that he recognizes in problems TE03 and TE04 the property of triangular inequality and that a triangle cannot be constructed. See figure 4, problem situations TE03 and TE04.

DATA ANALYSIS AND DISCUSSION OF RESULTS

For data analysis, an electronic sheet, Microsoft Excel type, was used to tabulate the responses of the professors; The test intended, in the first instance, for teachers to demonstrate their knowledge about the triangular inequality property; Table 1: expected analysis of the teacher includes the measures presented by each problem situation and if the property called triangular inequality is fulfilled for each of the figures and segments given.

The responses show that only one participant remembered the triangular inequality property; unfortunately, all the rest, 80% of the teachers did not remember that theorem and indicated that a triangle could be built with the segments of the problem situations TE03 and TE04.

The answer is wrong because the sum of the length of the two short sides is equal to the length of the longest side and therefore it is impossible to build triangles with them. Those professors assumed that it is enough to have three segments to build a triangle and even classified it as equilateral, isosceles, or scalene by the measures given. Special attention deserves the teacher who answered that the triangle cannot be built in the problem situation TE02 and indicates that data is missing, it is not based on the study theorem, but rather on the perception of the participant. See table 2 with the detail of the results.

Regarding the identification of the triangle based on the measurement of the sides or the segments that were presented to them, in TE01 all the teachers correctly identified the corresponding triangle; however, in TE02, 80% of the participants correctly indicate that we are building a scalene triangle; in TE05 and TE06 also 80% of the participants correctly identified the corresponding triangle in relation to the sides; in the identification of the triangle based on the angles, 60% of the teachers correctly identify TE02 as a right triangle because they remember the property a2 + b2 = c2; See Figure 5 for an extract of the problem situation taken from the questionnaire.Of the 5 participants, only the teacher with the most experience in teaching mathematics tried to make drawings, based on them he tried to identify if the figure turned out to be a right triangle; he mentally performed the operations and associated the measurements 6, 8 and 10 with the triangle with sides 3, 4 and 5 that the books present as a prototypical figure; In addition, the teacher who makes figures with her fingers is the one who remembered the property of triangular inequality, see in figure 6 teachers who analyze the problem situation.

Situation problem	Extent side a	Extent side b	Extent c side	*Satisfies triangle inequality*
TE01	10	10	10	*Yeah*
TE02	6	8	10	*Yeah*
TE03	4	6	10	*NO*
TE04	5	5	10	*NO*
TE05	5	6	10	*Yeah*
TE06	6	6	10	*Yeah*

Table1 *Correct answers expected from the teacher*

Detail of results or answers given by the participants

Problem situation	lets build triangle		Type of triangle by its sides			Type of triangle by its angles		
	YES	NO	Equilateral	Isosceles	Scalene	Acute angle	Rectangle	Obtuse angle
TE01	5	0	5	0	0	3	0	2
TE02	4	1	0	0	4	1	3	0
TE03	4	1	0	0	4	1	2	1
TE04	4	1	1	3	0	1	1	2
TE05	5	0	0	1	4	3	1	1
TE06	5	0	1	4	0	2	2	1

TE02	6	8	10	*Yeah*
TE03	4	6	10	*NO*
TE04	5	5	10	*NO*
TE05	5	6	10	*Yeah*
TE06	6	6	10	*Yeah*

Table2 *Detail of results or answers given by the participants*

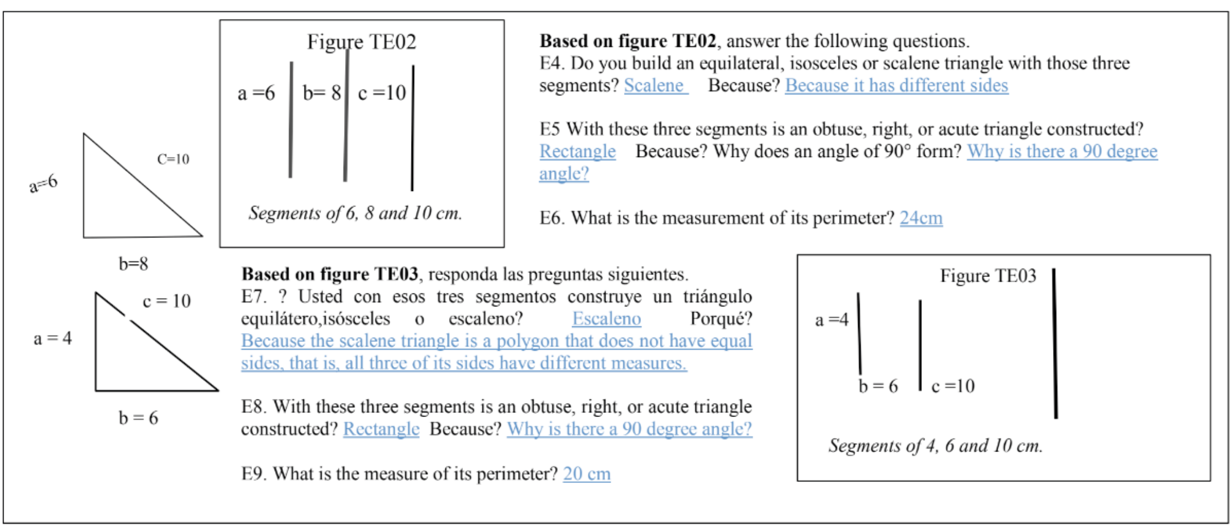

Figure 5 *An extract of the problem situation taken from the questionnaire*

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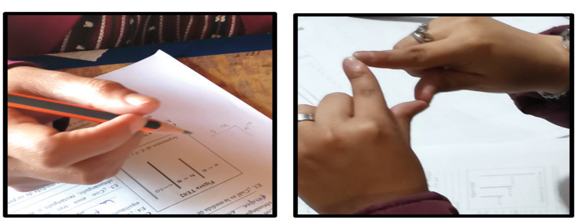
When observing the development of the teachers, the forgetfulness of the triangular inequality property is evident, the excess of prototypical problems in the course texts for the Pythagorean theorem makes it difficult to identify the right triangle in different positions, they do not apply this theorem to identify to the unconstructed right triangle, less to recognize the acute and obtuse angles. It is shown in figure 5 that the teacher almost proportionally reduces the triangle of problem TE02 with measures 6, 8 and 10 to match the figure of TE03; the triangular inequality goes unnoticed and neither does the Pythagorean theorem apply to verify whether or not it is a rectangle.

What is striking about the study, in addition to the forgetfulness of the triangular inequality, is the little application of the Pythagorean theorem in all the participants; only the oldest teacher dared to make drawings of triangles that matched his prototypical figures. Regarding the comprehensive management of the Pythagorean theorem, Rudi et al. (2020) "students experienced difficulties in understanding the definition, describing symbols or notations of mathematical objects, and interpreting mathematical objects in the form of a procedure to solve mathematical problems" (p. 14).



6a: Participant drawing pictures

6b: participant using their fingers

Figure 6 *Teachers analyzing the problem situation*

CONCLUSIONS AND RECOMMENDATIONS

The research question: What are the problems that high school mathematics teachers show in relation to triangular inequality? The answer is based on the data and their analysis; In the first place, the forgetfulness of the teachers is highlighted; they do not remember or are unaware of the triangular inequality property, so it is enough to indicate three segments, whatever their measurements, to build a triangle according to them and this error is replicated in the classroom. The second conclusion is that teachers do not apply the Pythagorean theorem to identify whether they will build an acute, right, or obtuse triangle with three segments.

It is recommended that teachers promote activities that allow the construction of triangles, not only use those already drawn in the texts, and that they include in their classes the use of concrete material in the construction of this figure. They are invited to extend the use of the Pythagorean theorem to identify different types of triangles, not just circumscribe it to the right triangle. They are invited to use technology in teaching the triangle, but always after using the concrete material and contextualized to different situations that arise in their communities.

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