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LOGIC AND POSSIBILITIES TO ANSWER THE QUESTION: WHAT IS MATHEMATICS?

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Abstract: This article derives from an experience carried out in the classroom, in order to answer some questions relevant to the usefulness of mathematical knowledge. It brings a suggestion for the Introduction to Reflection on "What is Mathematics", its applications, its logical structure, its functionality as a language. It starts with this questioning and ends with the exploration of a problem, evolving from a case of trial and error to a logical-combinational way of solving certain problem situations. In the analysis and elaboration of the solution, I approach both basic numerical/operational, as well as exploring concepts/characteristics of Simple Series, as is the case of Arithmetic Progressions, all without losing sight of the investigative characteristics that mathematics provides us, depending on how you approach it. As a contribution to this part, I work with some ideas on Mathematical Research in the Classroom, from Mathematics teachers: João Pedro da Ponte, Joana Brocardo and Hélia Oliveira. I also talk about: the Language Obstacle, the logical structure that a language must have, the symbolic completeness required to efficiently communicate an idea, the interpretation of facts in Problem Solving, the analysis of data and its properties and the generalization of the possible results. To do so, I will dialogue with the teachers: Júlio César de Melo e Sousa (Malba Tahan) and Luiz Carlos Pais, in addition to a quick conversation with the psychologist, anthropologist and sociologist Carlos Rodrigues Brandão about Education. I conclude, justifying algebra and the value of generalizations, with a very simple example: Mathemagic, to paraphrase Walt Disney.

Keywords: Education, Mathematics, Research, Logic, Possibilities.

INTRODUCTION

Before sketching the initial ideas that led me to carry out this experiment, I remember that Malba Tahan, in his most famous book, has already left us a very convincing way of answering the ever-present questions about mathematical knowledge: What is it and what is mathematics for? In his book, Tahan begins his narrative by talking about envy, one of the characteristics and a certain vizier of the court of Al-Motacem named Nahum Ibn-Nahum. Seeing the prestige of Beremiz (the man who calculated) grow before the caliph, Nahum "decided to embarrass my talented friend and place him in a ridiculous and false situation. So it was that he approached the king and said to him, distilling the words" (TAHAN, 2001, p. 104-105):

> – I have just observed, O Emir of the Believers, that the Persian calculator, our guest this afternoon, is expert in counting elements or figures in a collection. He counted the five hundred or so words written on the wall of the salon, cited two friendly numbers, talked about the difference (64 which is cube and square) and ended up counting, one by one, the fringes of the skirts of the beautiful dancers.
>
> We would be badly served if our mathematicians were willing to take care of such puerile things, with no practical use of any kind. Really! What is the use of knowing if there are, in the verses that enrapture us, 220 or 284 words and if these numbers are friends or not? The concern of those who admire a poet is not to count the lyrics of the verses. [...] Nor do we care whether the dress of this beautiful and graceful dancer has 312, 309 or 1,000 fringes. All this is ridiculous and of very little interest to men of feeling who cultivate Beauty and Art.

The vizier ends his speech, not at all friendly, with the ideas that, generally, populate the thoughts of those unaware of the most common origin of scientific construction: the simple pleasure in investigating facts, coherences, characteristics, conjectures:

> Human ingenuity, supported by science, must be devoted to solving the great problems of Life. The sages – inspired by Allah the Exalted – did not erect the dazzling edifice of Mathematics so that this noble science could have the application that the Persian calculator wants to attribute to it. It seems to me, therefore, a crime to reduce the science of a Euclid, an Archimedes or a wonderful Omar Khayyãm (Allah have him in his glory!) to that miserable situation of numerical evaluator of things and beings. We are interested, therefore, in seeing this calculator apply the theories (which he claims to have) in the solution of problems of real use, that is, problems that relate to the needs and claims of everyday life! (TAHAN, 2001, p. 105)

In a way, we are periodically challenged with statements similar to this one, leading to the erroneous thought that science is obliged to justify every theory through its immediately practical applicability. To this the author replied:

> – The doctors and ulema, O King of the Arabs, do not ignore that Mathematics arose with the awakening of the human soul; but it did not arise for utilitarian purposes. It was the urge to solve the mystery of the Universe, before which man is a mere grain of sand, that gave him his first impulse. Its true development resulted, first of all, from the effort to penetrate and understand the Infinite. And even today, after we have spent centuries trying, in vain, to remove the thick wake, even today it is the search for the Infinite that takes us forward. The material progress of men depends on the abstract or scientific research of the present, and it will be to the men of science who work it for purely scientific ends, without any intention of applying its doctrines, that humanity will be indebted in future times (TAHAN, 2001, p. 106).

The construction of scientific thinking has, among its characteristics, the sequentiality of knowledge, that is, a new theory must have as input a pre-existing one. The author justifies:

> - When the mathematician performs his calculations, or seeks new relationships between numbers, he does not seek the truth for utilitarian ends. To cultivate science for its practical, immediate use is to distort the soul of science itself!
>
> Will the theory studied today, and which seems useless to us, have applications in the future? [...] It is quite possible that today's theoretical investigations will provide, within a thousand or two thousand years, precious resources for practice.
>
> It is also necessary not to forget that mathematics, in addition to the objective of solving problems, calculating areas and measuring volumes, has much higher purposes.
>
> As it has a high value in the development of intelligence and reasoning, Mathematics is one of the safest ways through which we can lead man to feel the power of thought, the magic of the spirit (TAHAN, 2001, p. 106-107).

And he ends his justifications, considering the beauty of doing science, comparing it with the elevation of the spirit that welcomes us when we contemplate nature and see in it the scientific language. This is the goal: to translate what happens in the world into analytical symbology. Science in life!

> Mathematics is, after all, one of the eternal truths, and, as such, it produces the elevation of the spirit - the same elevation that we feel when contemplating the great spectacles of Nature, through which we feel the presence of God, Eternal and Omnipotent! There is, therefore, O illustrious Vizier Nahum Ibn-Nahum, as I have said, a small error on your part. I count the lines of a poem, calculate the height of a star, assess the number of fringes, measure the area of a country, or the strength of a torrent – I apply, in short, algebraic formulas and geometric principles – without worrying about the laurels. that I can take from my calculations and studies! Without dreams and fantasy, science is wealthy. It's dead science! (TAHAN, 2001, p. 107)

Finally, the king who witnessed this dialogue, impressed by Beremiz's eloquence, raised his right hand to him and said: "- The theory of the dreaming scientist won and will always win the crude immediacy of the ambitious without a philosophical ideal!" (TAHAN, 2001, p. 108)

These cuts, in my view, would justify the fact that we study and develop science. However, I present to the students the philosophical justification (this one, brilliantly written by Professor Júlio César de Melo e Sousa), but I also present a theoretical justification, focused on mathematical knowledge in practice. I present a common mathematical problem and we turn it into an investigative object. We start by assessing the existence of patterns. If they are identified, conjectures are elaborated, which will be investigated. If found, for their validity, they must be generalized. Let's go to the method.

METHODOLOGY

According to law number 9.394, of December 20, 1996, in its first article, "education encompasses the formative processes that are developed in family life, in human coexistence, at work, in teaching and research institutions, in social and social movements". civil society organizations and cultural events". Still, in its second paragraph, we see that "school education must be linked to the world of work and social practice" (BRASIL, 1996). Even though it is a recent law, considering, mainly, the previous one (law nº 4.024, of December 20, 1961), these principles accompany me since the first classes taught in Elementary School II, at the time, 5th. the 8th. Series; the year was 1988. Two questions have always bothered me, considering what Education must be: the first, itself: What is Education? The second: What is Mathematics? Reflecting on the first, according to Brandão (1987, p.10):

> [...] Education can exist freely and, among all, it can be one of the ways that people create to make *common*, as knowledge, as an idea, as a belief, what is *community* as a good, as work or as life. [...] Education is, like others, a fraction of the *way of life* of the social groups that create and recreate it, among so many other inventions of their culture, in their society.

As an art of creating and recreating, I began to imagine how education must act as an intermediary for the learning of mathematics. With this principle, I began my investigation into the concept of mathematics and what it is for in terms of work, in terms of life.

When I started working with teacher training in the 90s, I began to notice a certain difficulty in justifying mathematical knowledge. What will this serve in my life, teacher? This question, initially assigned to elementary school students, was also insistent in the academic environment. The need for *why* brings with it the *why* and *what we* must teach. With that in mind, one way I found to start these discussions with future teachers was the following:

First the question: What is Mathematics? At this point, I consider mathematics as a language used to express, represent and solve problems involving calculations. As a language, it cannot just be expressed symbolically. It must carry in its essence a logic, at least, organizational for an effective communication of its ideas. An example is the following sequence: **25,R 90$180.** What does this grouping of symbols mean? What is the idea communicated? We don't know exactly, do we? That's because your organization is lacking logic. When we use the same symbols as follows: **R$ 25981.00** now yes, we have the exact idea of what we are talking about. This is a first obstacle faced in learning mathematics, the language obstacle. In addition to this perspective, we still have the question of meanings. "It is the case of a student who stated that the square did not have any property, because the meaning attributed by him to the word *property* would be a house, a land or a motorcycle, as people express themselves in the context of their family life" (PARENTS), 2006, pp. 77-78).

This mathematical characteristic is evident when compared to our mother tongue. There is a linguistic puzzle that illustrates this logical, meaningful and complementary approach between languages. The word complementary, in this case, refers to the lack of some symbol in your communication. Logic alone is not enough. The symbology must be complete. Let's go to the puzzle: in the clipping "MARIA TAKES A BATH BECAUSE HER MOTHER SAID SHE BRING ME THE TOWEL", what is the message to be conveyed? There is logic in the ordering of symbols, but it seems that something is missing, doesn't it? And yes, it is missing: two commas and a period are missing, without which, it is difficult to conclude the true message to which the clipping refers. The solution makes clear the complexity of meanings. Many cannot solve this puzzle due to this linguistic property, made even more difficult by the richness of meanings that some words have in our Portuguese language. The difficulty lies in the analysis of the word "SUA". Here, she is not a possessive pronoun but a verb. Then we will have:

MARIA TAKES A BATH BECAUSE IT'S HER. MOM, SHE SAID, BRING ME THE TOWEL.

The Second Step is to show that Mathematics is a language that works with Logic and Possibilities. For that, we use the example of solving the Magic Square 3 x 3. For those who don't know, the Magic Square is a puzzle where different numbers are available whose sum vertically, horizontally and diagonally is always the same. In the case of the classic 3 x 3, the numbers to be used are from 1 to 9 and the constant sum (vertically, horizontally and diagonally) is 15. In a first contact, the method used to solve this type of hobby It is the trial and error method. The numbers are randomly distributed in the square and the crowd begins so that the rules are met as if by magic. However, we can solve this problem in a logical way, checking the Possibilities of solution. To illustrate the problem, we work with the following square:

The problem: Considering the 3 x 3 Magic Square above, fill it with the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, so that the horizontal, vertical, and diagonal sums are always 15. The problem can be redefined as follows: with the numbers 1 to 9, find the possible additions, with three distinct parts, whose sum is always 15. The possibilities are represented in the following table:

1 + 9 + 5	3 + 7 + 5	1 + 8 + 6	3 + 8 + 4
2 + 8 + 5	4 + 6 + 5	2 + 7 + 6	4 + 9 + 2

The logic is in the properties of the problem:

1st logic) We have eight Additions required (three vertically, three horizontally, and two diagonally). So we have eight possible additions. When we find them, we can end this phase of the solution;

2nd logic) We have three types of parcels among the possible additions: those that will be in the "corner houses"; the ones that will be in the "lateral means" and the one located in the center of the square. The latter must appear in four possible additions (one vertical, one horizontal and two diagonals); the "sideways" plots appear in two possible additions (one vertical and one horizontal) and the "corners" plots appear in three additions (one vertical, one diagonal and one horizontal). The possibilities are illustrated in the following table:

Houses	installments
“from the corners”	2, 4, 6, 8
center	5
“side means”	1, 3, 7, 9

Considering the Logic and the Possibilities to fill the square, we have:

2	**7**	**6**
9	**5**	**1**
4	**3**	**8**

The solution of the problem is already determined, however, with these resolution criteria, we conclude something very interesting at the end of the analysis of the problem.

RESULTS AND DISCUSSION

If the sequence of numbers is in the same ratio, the reasoning does not change, both in terms of the number of possible additions and the criteria for filling the square. To illustrate, we can now consider the numbers: 21, 22, 23, 24, 25, 26, 27, 28 and 29. The possible additions are illustrated in the following table:

21 + 29 + 25	21 + 28 + 26
22 + 28 + 25	22 + 27 + 26
23 + 27 + 25	22 + 29 + 24
24 + 26 + 25	23 + 28 + 24

The possibilities are illustrated in the following table:

Houses	installments
“from the corners”	22, 24, 26, 28
center	25
“side means”	21, 23, 27, 29

Considering the Logic and the Possibilities to fill the square, we have:

22	**27**	**26**
29	**25**	**21**
24	**23**	**28**

The constant sum, in this case, becomes 75. These logics lead us to the interesting outcome of this puzzle. As the method is valid for any numerical sequence with constant ratio, it is valid for any Arithmetic Progression with nine terms. So, if we consider the Sequence: 3, 6, 9, 12, 15, 18, 21, 24 and 27, we will have:

6	**21**	**18**
27	**15**	**3**
12	**9**	**24**

a) If the ratio is constant, the number of additions and the possibilities remain the same and

b) Therefore, the possible answers will also be the same (considering that we can rotate or invert the position of the numbers) and, therefore, the positioning of the numbers will also be the same. Therefore, for all Arithmetic Progression, the following comparative table is valid (see that it does not depend on the numbers and the ratio, as long as it is the same throughout the Numerical Sequence):

	positioning								
Initial Numbers	1	2	3	4	5	6	7	8	9
Sequence 02	21	22	23	24	25	26	27	28	29
Sequence 03	3	6	9	12	15	18	21	24	27
Sequence 04 (Extra)	103	106	109	112	115	118	121	124	127
Sequence 05 (Extra)	52	56	60	64	68	72	76	80	84

Considering the placements to fill the Square, just compare with the initial Magic Square. Thus, we will have the following solutions:

106	121	118
127	115	103
112	109	124

56	76	72
84	68	52
64	60	80

In the first Square, the constant sum is 345 and in the second, the sum is 204.

So if we must, in the end, just compare with the initial solution, we can conclude that:

a) The sums of the Arithmetic Progression terms are always constant. In fact, the solution will always be of the type:

a_2	at_7	at_6
at_9	at_5	at_1
at_4	at_3	at_8

Therefore, considering the eight possible addition possibilities (three horizontally, three vertically and two diagonally), we have that (Remember that: in Arithmetic Progression, every term will be of the type: $\mathbf{A_n = A_1 + (n - 1). r}$):

$$A_2 + A_7 + A_6 = A_2 + A_5 + A_8 = A_2 + A_9 + A_4 = \ldots \rightarrow A_1 + r + A_1 + 6r + A_1 + 5r = \boxed{3.A_1 + 12.\,r}$$

Where the index indicates the position of the number in the Arithmetic Progression and the r indicates the rate of progression. Thus, A_2 refers to the second term of the sequence and the ratio is the difference between the terms. In the case of the sequence: 3, 6, 9, 12, 15, 18, 21, 24 and 27, the ratio “r” will be 3;

b) The constant sum (S_c) will be equal to 3. A_1+12. r, that is:

$$\mathbf{S_c = 3.\,A_1 + 12.r = 3.(A_1 + 4.r) = 3.A_5.}$$

So, in order to know the constant Sum, we just multiply the fifth term of the Progression by 3. As an example, let the sequence be 4106, 4112, 4118, 4124, 4130, 4136, 4142, 4148 and 4154. What will be the solution and what will be the constant sum? The constant sum will be: Sc = 3. 4130 = 12390. The solution will be:

4112	4142	4136
4154	4130	4106
4124	4118	4148

The 3 x 3 Magic-Square example showed a good practical application of the work with numerical sequences. It is a theory that involves, at first glance, terms and their positions in the sequence. If the latter is fixed and considering its own particularities, the Sequence receives special names, identifying its main characteristic. In our case, we work with Arithmetic Progressions, where the difference between their terms is always the same. However, we can also come across cases that only take into account the elements and their positions. This is the case of an ancient mathematical “magic”, where “guessing” works as follows:

From a complete deck, that is, with 52 cards, consider 27 of them. Ask someone to choose any of the cards and, without you looking, put it back in the deck and shuffle as many times as you like. This time, make three piles or columns of cards, in the same sequence, and ask the person to say which pile the chosen card is in. When identifying the pile, collect each pile or column, keeping the sequence of cards, placing the identified pile in the middle

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of the other two. Repeat the process three times. From the fourth time onwards, the "secret" card will be the 14th. dealt card, that is, it will be the fifth card of the middle pile or column. Of course, the person who chose the letter cannot know this final detail. This is the "magic" that makes you "guess" the chosen card. Let's go to Logic and Possibilities:

Let the random sequence of cards be: A_{1}, A_{2}, A_{3}, A_{4}, A_{5}, A_{6}, A_{7}, A_{8}, A_{9}, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}, A_{19}, A_{20}, A_{21}, A_{22}, A_{23}, A_{24}, A_{25}, A_{26}, A_{27}. Suppose the chosen card is in position A_{4}. Thus, in the first distribution of cards, we will have the following possibilities of piles or columns:

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{1}$	a $_{2}$	at $_{3}$
at $_{4}$	at $_{5}$	at $_{6}$
at $_{7}$	at $_{8}$	at $_{9}$
at $_{10}$	at $_{11}$	at $_{12}$
at $_{13}$	at $_{14}$	at $_{15}$
at $_{16}$	at $_{17}$	at $_{18}$
at $_{19}$	at $_{20}$	at $_{21}$
at $_{22}$	the $_{23}$	at $_{24}$
at $_{25}$	at $_{26}$	the $_{27}$

See that, at this first moment, the card is in the first column or first pile. We have two possibilities of putting the cards together again, remembering that the column where the card is located must come in the middle. So we will have:

POSSIBILITY OF SEQUENCE 01

A_{2}, A_{5}, A_{8}, A_{11}, A_{14}, A_{17}, A_{20}, A_{23}, A_{26}, A_{1}, **A_{4}**, A_{7}, A_{10}, A_{13}, A_{16}, A_{19}, A_{22}, A_{25}, A_{3}, A_{6}, A_{9}, A_{12}, A_{15}, A_{18}, A_{21}, A_{24}, A_{27}, or:

POSSIBILITY OF SEQUENCE 02

A_{3}, A_{6}, A_{9}, A_{12}, A_{15}, A_{18}, A_{21}, A_{24}, A_{27}, A_{1}, **A_{4}**, A_{7}, A_{10}, A_{13}, A_{16}, A_{19}, A_{22}, A_{25}, A_{2}, A_{5}, A_{8}, A_{11}, A_{14}, A_{17}, A_{20}, A_{23}, A_{26}.

Note that the chosen A_{4} occupies the same position in both possibilities, that is, the 11th. position in the new sequence. Distributing the cards again, we have:

POSSIBILITY OF SEQUENCE 01

Column or Mount 01	Column or Mount 02	Column or Mount 03
a $_{2}$	at $_{5}$	at $_{8}$
at $_{11}$	at $_{14}$	at $_{17}$
at $_{20}$	the $_{23}$	at $_{26}$
at $_{1}$	**at $_{4}$**	at $_{7}$
at $_{10}$	at $_{13}$	at $_{16}$
at $_{19}$	at $_{22}$	at $_{25}$
at $_{3}$	at $_{6}$	at $_{9}$
at $_{12}$	at $_{15}$	at $_{18}$
at $_{21}$	at $_{24}$	the $_{27}$

POSSIBILITY OF SEQUENCE 02

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{3}$	at $_{6}$	at $_{9}$
at $_{12}$	at $_{15}$	at $_{18}$
at $_{21}$	at $_{24}$	the $_{27}$
at $_{1}$	**at $_{4}$**	at $_{7}$
at $_{10}$	at $_{13}$	at $_{16}$
at $_{19}$	at $_{22}$	at $_{25}$
a $_{2}$	at $_{5}$	at $_{8}$
at $_{11}$	at $_{14}$	at $_{17}$
at $_{20}$	the $_{23}$	at $_{26}$

See that, in this second moment, the card is in the second column or second pile. We have two possibilities of putting the cards back together. So we will have:

POSSIBILITY OF SEQUENCE 01

$A_{2}, A_{11}, A_{20}, A_{1}, A_{10}, A_{19}, A_{3}, A_{12}, A_{21}, A_{5}, A_{14}, A_{23}, \mathbf{A_{4}}, A_{13}, A_{22}, A_{6}, A_{15}, A_{24}, A_{8}, A_{17}, A_{26}, A_{7}, A_{16}, A_{25}, A_{9}, A_{18}, A_{27},$ or:
$A_{8}, A_{17}, A_{26}, A_{7}, A_{16}, A_{25}, A_{9}, A_{18}, A_{27}, A_{5}, A_{14}, A_{23}, \mathbf{A_{4}}, A_{13}, A_{22}, A_{6}, A_{15}, A_{24}, A_{2}, A_{11}, A_{20}, A_{1}, A_{10}, A_{19}, A_{3}, A_{12}, A_{21}.$

POSSIBILITY OF SEQUENCE 02

$A_{3}, A_{12}, A_{21}, A_{1}, A_{10}, A_{19}, A_{2}, A_{11}, A_{20}, A_{6}, A_{15}, A_{24}, \mathbf{A_{4}}, A_{13}, A_{22}, A_{5}, A_{14}, A_{23}, A_{9}, A_{18}, A_{27}, A_{7}, A_{16}, A_{25}, A_{8}, A_{17}, A_{26},$ or:
$A_{9}, A_{18}, A_{27}, A_{7}, A_{16}, A_{25}, A_{8}, A_{17}, A_{26}, A_{6}, A_{15}, A_{24}, \mathbf{A_{4}}, A_{13}, A_{22}, A_{5}, A_{14}, A_{23}, A_{3}, A_{12}, A_{21}, A_{1}, A_{10}, A_{19}, A_{2}, A_{11}, A_{20}.$

Note that the chosen A_{4} occupies the same positions in the four possibilities, that is, the 13th. position in the new sequences. Distributing the cards again, we have:

POSSIBILITY OF SEQUENCE 01

Column or Mount 01	Column or Mount 02	Column or Mount 03
a $_{2}$	at $_{11}$	at $_{20}$
at $_{1}$	at $_{10}$	at $_{19}$
at $_{3}$	at $_{12}$	at $_{21}$
at $_{5}$	at $_{14}$	the $_{23}$
at $_{4}$	at $_{13}$	at $_{22}$
at $_{6}$	at $_{15}$	at $_{24}$
at $_{8}$	at $_{17}$	at $_{26}$
at $_{7}$	at $_{16}$	at $_{25}$
at $_{9}$	at $_{18}$	the $_{27}$

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{8}$	at $_{17}$	at $_{26}$
at $_{7}$	at $_{16}$	at $_{25}$
at $_{9}$	at $_{18}$	the $_{27}$
at $_{5}$	at $_{14}$	the $_{23}$
at $_{4}$	at $_{13}$	at $_{22}$
at $_{6}$	at $_{15}$	at $_{24}$
a $_{2}$	at $_{11}$	at $_{20}$
at $_{1}$	at $_{10}$	at $_{19}$
at $_{3}$	at $_{12}$	at $_{21}$

POSSIBILITY OF SEQUENCE 02

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{3}$	at $_{12}$	at $_{21}$
at $_{1}$	at $_{10}$	at $_{19}$
a $_{2}$	at $_{11}$	at $_{20}$
at $_{6}$	at $_{15}$	at $_{24}$
at $_{4}$	at $_{13}$	at $_{22}$
at $_{5}$	at $_{14}$	the $_{23}$
at $_{9}$	at $_{18}$	the $_{27}$
at $_{7}$	at $_{16}$	at $_{25}$
at $_{8}$	at $_{17}$	at $_{26}$

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{9}$	at $_{18}$	the $_{27}$
at $_{7}$	at $_{16}$	at $_{25}$
at $_{8}$	at $_{17}$	at $_{26}$
at $_{6}$	at $_{15}$	at $_{24}$
at $_{4}$	at $_{13}$	at $_{22}$
at $_{5}$	at $_{14}$	the $_{23}$
at $_{3}$	at $_{12}$	at $_{21}$
at $_{1}$	at $_{10}$	at $_{19}$
a $_{2}$	at $_{11}$	at $_{20}$

See that, in this third moment, the card is in the first column or first pile. We have eight possibilities to put the cards back together. So we will have:

POSSIBILITY OF SEQUENCE 01

$A_{11}, A_{10}, A_{12}, A_{14}, A_{13}, A_{15}, A_{17}, A_{16}, A_{18}, A_{2}, A_{1}, A_{3}, A_{5}, \mathbf{A_{4}}, A_{6}, A_{8}, A_{7}, A_{9}, A_{20}, A_{19}, A_{21}, A_{23}, A_{22}, A_{24}, A_{26}, A_{25}, A_{27},$ or:

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A_{20}, A_{19}, A_{21}, A_{23}, A_{22}, A_{24}, A_{26}, A_{25}, A_{27}, A_{2}, A_{1}, A_{3}, A_{5}, $\mathbf{A_{4}}$, A_{6}, A_{8}, A_{7}, A_{9}, A_{11}, A_{10}, A_{12}, A_{14}, A_{13}, A_{15}, A_{17}, A_{16}, A_{18}, or:
A_{17}, A_{16}, A_{18}, A_{14}, A_{13}, A_{15}, A_{11}, A_{10}, A_{12}, A_{8}, A_{7}, A_{9}, A_{5}, $\mathbf{A_{4}}$, A_{6}, A_{2}, A_{1}, A_{3}, A_{26}, A_{25}, A_{27}, A_{23}, A_{22}, A_{24}, A_{20}, A_{19}, A_{21}, or:
A_{26}, A_{25}, A_{27}, A_{23}, A_{22}, A_{24}, A_{20}, A_{19}, A_{21}, A_{8}, A_{7}, A_{9}, A_{5}, $\mathbf{A_{4}}$, A_{6}, A_{2}, A_{1}, A_{3}, A_{17}, A_{16}, A_{18}, A_{14}, A_{13}, A_{15}, A_{11}, A_{10}, A_{12}.

POSSIBILITY OF SEQUENCE 02

A_{12}, A_{10}, A_{11}, A_{15}, A_{13}, A_{14}, A_{18}, A_{16}, A_{17}, A_{3}, A_{1}, A_{2}, A_{6}, $\mathbf{A_{4}}$, A_{5}, A_{9}, A_{7}, A_{8}, A_{21}, A_{19}, A_{20}, A_{24}, A_{22}, A_{23}, A_{27}, A_{25}, A_{26}, or:
A_{21}, A_{19}, A_{20}, A_{24}, A_{22}, A_{23}, A_{27}, A_{25}, A_{26}, A_{3}, A_{1}, A_{2}, A_{6}, $\mathbf{A_{4}}$, A_{5}, A_{9}, A_{7}, A_{8}, A_{12}, A_{10}, A_{11}, A_{15}, A_{13}, A_{14}, A_{18}, A_{16}, A_{17}, or:
A_{18}, A_{16}, A_{17}, A_{15}, A_{13}, A_{14}, A_{12}, A_{10}, A_{11}, A_{9}, A_{7}, A_{8}, A_{6}, $\mathbf{A_{4}}$, A_{5}, A_{3}, A_{1}, A_{2}, A_{27}, A_{25}, A_{26}, A_{24}, A_{22}, A_{23}, A_{21}, A_{19}, A_{20}, or:
A_{27}, A_{25}, A_{26}, A_{24}, A_{22}, A_{23}, A_{21}, A_{19}, A_{20}, A_{9}, A_{7}, A_{8}, A_{6}, $\mathbf{A_{4}}$, A_{5}, A_{3}, A_{1}, A_{2}, A_{18}, A_{16}, A_{17}, A_{15}, A_{13}, A_{14}, A_{12}, A_{10}, A_{11}.

This time, the chosen A_4 occupies the same position in the eight possibilities, that is, the 14th. position in the new sequences. Distributing the cards again, we have:

ONE OF THE SEQUENCE POSSIBILITIES 01

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{11}$	at $_{10}$	at $_{12}$
at $_{14}$	at $_{13}$	at $_{15}$
at $_{17}$	at $_{16}$	at $_{18}$
a $_{2}$	at $_{1}$	at $_{3}$
at $_{5}$	**at $_{4}$**	at $_{6}$
at $_{8}$	at $_{7}$	at $_{9}$
at $_{20}$	at $_{19}$	at $_{21}$
the $_{23}$	at $_{22}$	at $_{24}$
at $_{26}$	at $_{25}$	the $_{27}$

ONE OF THE SEQUENCE POSSIBILITIES 02

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{12}$	at $_{10}$	at $_{11}$
at $_{15}$	at $_{13}$	at $_{14}$
at $_{18}$	at $_{16}$	at $_{17}$
at $_{3}$	at $_{1}$	a $_{2}$
at $_{6}$	**at $_{4}$**	at $_{5}$
at $_{9}$	at $_{7}$	at $_{8}$
at $_{21}$	at $_{19}$	at $_{20}$
at $_{24}$	at $_{22}$	the $_{23}$
the $_{27}$	at $_{25}$	at $_{26}$

By induction, we can imagine what the position of the chosen card will now be, in all eight possibilities: the 14th. Position. To verify this, let us consider the two possibilities of each previous table, where we will have:

A_{11}, A_{14}, A_{17}, A_{2}, A_{5}, A_{8}, A_{20}, A_{23}, A_{26}, A_{10}, A_{13}, A_{16}, A_{1}, $\mathbf{A_{4}}$, A_{7}, A_{19}, A_{22}, A_{25}, A_{12}, A_{15}, A_{18}, A_{3}, A_{6}, A_{9}, A_{21}, A_{24}, A_{27}, or:
A_{12}, A_{15}, A_{18}, A_{3}, A_{6}, A_{9}, A_{21}, A_{24}, A_{27}, A_{10}, A_{13}, A_{16}, A_{1}, $\mathbf{A_{4}}$, A_{7}, A_{19}, A_{22}, A_{25}, A_{11}, A_{14}, A_{17}, A_{2}, A_{5}, A_{8}, A_{20}, A_{23}, A_{26}, or:
A_{12}, A_{15}, A_{18}, A_{3}, A_{6}, A_{9}, A_{21}, A_{24}, A_{27}, A_{10}, A_{13}, A_{16}, A_{1}, $\mathbf{A_{4}}$, A_{7}, A_{19}, A_{22}, A_{25}, A_{11}, A_{14}, A_{17}, A_{2}, A_{5}, A_{8}, A_{20}, A_{23}, A_{26}, or:
A_{11}, A_{14}, A_{17}, A_{2}, A_{5}, A_{8}, A_{20}, A_{23}, A_{26}, A_{10}, A_{13}, A_{16}, A_{1}, $\mathbf{A_{4}}$, A_{7}, A_{19}, A_{22}, A_{25}, A_{12}, A_{15}, A_{18}, A_{3}, A_{6}, A_{9}, A_{21}, A_{24}, A_{27}.

From that moment on, this card will always occupy this position. This becomes clear when we verify that the Sequences are repeated two by two. In fact:

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{11}$	at $_{14}$	at $_{17}$
a $_{2}$	at $_{5}$	at $_{8}$

at $_{20}$	the $_{23}$	at $_{26}$
at $_{10}$	at $_{13}$	at $_{16}$
at $_{1}$	**at $_{4}$**	at $_{7}$
at $_{19}$	at $_{22}$	at $_{25}$
at $_{12}$	at $_{15}$	at $_{18}$
at $_{3}$	at $_{6}$	at $_{9}$
at $_{21}$	at $_{24}$	the $_{27}$

Column or Mount 01	Column or Mount 02	Column or Mount 03
at $_{12}$	at $_{15}$	at $_{18}$
at $_{3}$	at $_{6}$	at $_{9}$
at $_{21}$	at $_{24}$	the $_{27}$
at $_{10}$	at $_{13}$	at $_{16}$
at $_{1}$	**at $_{4}$**	at $_{7}$
at $_{19}$	at $_{22}$	at $_{25}$
at $_{11}$	at $_{14}$	at $_{17}$
a $_{2}$	at $_{5}$	at $_{8}$
at $_{20}$	the $_{23}$	at $_{26}$

Arranging the possible sequences, we have:

$A_{11}, A_{2}, A_{20}, A_{10}, A_{1}, A_{19}, A_{12}, A_{3}, A_{21}, A_{14}, A_{5}, A_{23}, A_{13}, \mathbf{A_{4}}, A_{22}, A_{15}, A_{6}, A_{24}, A_{17}, A_{8}, A_{26}, A_{16}, A_{7}, A_{25}, A_{18}, A_{9}, A_{27},$ or:

$A_{17}, A_{8}, A_{26}, A_{16}, A_{7}, A_{25}, A_{18}, A_{9}, A_{27}, A_{14}, A_{5}, A_{23}, A_{13}, \mathbf{A_{4}}, A_{22}, A_{15}, A_{6}, A_{24}, A_{11}, A_{2}, A_{20}, A_{10}, A_{1}, A_{19}, A_{12}, A_{3}, A_{21},$ or:

$A_{12}, A_{3}, A_{21}, A_{10}, A_{1}, A_{19}, A_{11}, A_{2}, A_{20}, A_{15}, A_{6}, A_{24}, A_{13}, \mathbf{A_{4}}, A_{22}, A_{14}, A_{5}, A_{23}, A_{18}, A_{9}, A_{27}, A_{16}, A_{7}, A_{25}, A_{17}, A_{8}, A_{26},$ or:

$A_{18}, A_{9}, A_{27}, A_{16}, A_{7}, A_{25}, A_{17}, A_{8}, A_{26}, A_{15}, A_{6}, A_{24}, A_{13}, \mathbf{A_{4}}, A_{22}, A_{14}, A_{5}, A_{23}, A_{12}, A_{3}, A_{21}, A_{10}, A_{1}, A_{19}, A_{11}, A_{2}, A_{20}.$

Again we have the 14th. position, as we wanted to verify.

FINAL CONSIDERATIONS

The choice of the "Magic-Square" hobby was not random. The basis of its solution is concentrated on fundamental operations, considered the principle of abstraction and assimilation of mathematical knowledge. According to Ponte, Brocardo and Oliveira (2013, p. 55),

> The concept of number occupies a prominent place in school mathematics. Developing number sense, that is, acquiring a global understanding of numbers and operations and using it flexibly to analyze situations and develop useful strategies for dealing with numbers and operations, is a central objective of learning mathematics. Numerical investigations contribute decisively to developing this global understanding of numbers and operations, as well as important mathematical skills such as formulating and testing conjectures and looking for generalizations. Students can carry out small investigations that lead to the discovery of facts, properties, and relationships between sets of numbers. They can investigate aspects related to decimals, divisors or multiples of different numbers. They can also explore numerical sequences, discovering numerical relationships and progressively apprehending the idea of a variable. They can also establish connections between numbers and geometry.

Also according to the same authors, "for professional mathematicians, to investigate is to discover relationships between known or unknown mathematical objects, seeking to identify their respective properties" (PONTE, BROCARDO and OLIVEIRA, 2013, p. 13). These considerations led me to look for an elementary starting point but, at the same time, one that required a logical reasoning and a slightly more robust mathematical Perception. Working with teacher training, I try to consider visually trivial situations, however, with a certain elegance of reasoning. Problems with these characteristics corroborate the development of creativity, autonomy, analysis, and a generalizing look. These requirements are lacking, for the most part, in the current training of new teachers in the area of Mathematics.

However, we cannot fail to consider Pais' observations (2006, p. 13), when they say that:

> There are several arguments used to defend the existence of school mathematics. From kindergarten to high school, this discipline has been considered capable of contributing to the intellectual formation of the student. However, this argument, by itself, does not provide any guarantee of achievement of the foreseen objectives. There is a great distance between what can be accomplished in terms of objectives and the actual realization of the possible. Overcoming this distance certainly depends on many variables: teacher training, redefinition of methods, expansion of current research fields, creation and diversification of strategies, incorporation of the qualitative use of digital technologies and, still, a good dose of availability to reverse conceptions. hardened by time.

Therefore, planning, diversifying, contextualizing, exemplifying, exploring, still make a lot of difference when trying to teach, because this verb, a long time ago, is no longer just passing on knowledge, transmitting, instructing. "The nature of man, in his double body and spiritual structure, creates special conditions for the maintenance and transmission of his particular form and requires physical and spiritual organizations, to which we give the name of education" (BRANDÃO, 1987, p. 14). Today, therefore, [...] "students need to know how to critically interpret the way numbers are used in everyday life and the school must seek to develop this type of competence " (PONTE, BROCARDO and OLIVEIRA, 2013, p. 70). After all, training for life requires more than repeating, imitating, following algorithms, manipulating processes. It demands criticality, creativity, autonomy, perspectives, logic in its various possibilities.

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