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THE CONSTRUCTION OF MATHEMATICAL KNOWLEDGE: CURIOSITIES IN A LACON AND PHILOSOPHICAL PERSPECTIVE

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Abstract: When he stopped being a nomad and started to live a principle of society, man began to feel the need to count. The first idea gave rise to one-to-one correspondence (basis for working with concepts involving function). The fingers of the hands, stones (from the Latin, calculus) become instruments used in this correspondence, whose decimal traces are preserved and considered to this day. However, groups of stones have ephemeral characteristics that make it impossible to preserve numerical information in the long term. The first innovative idea was to register values through marks on bones and sticks. Thus begins a language that was developed within principles, whose objective was to facilitate the work of the man who calculated (even without him knowing it). "The tendency of language to develop from the concrete to the abstract can be seen in many of the measures of length in use today. A horse's height is measured in "spans" and the words "foot" and "ell" (elbow, elbow) also derive from body parts" (BOYER, 1996, p. 04). All this contributed to the development of mathematics, transforming it into something much bigger than just counting and measuring. The origin of the first indications of a structure, which would later be called mathematics, is lost in times without records. Sometimes it is due to practical necessity (in Herodotus' view - the "rope-drawers" of ancient Egypt, for example), sometimes to priestly and ritual leisure (in Aristotle's view). Therefore, our chronological observation will have as a principle to highlight important points according to personal motivations, arranged on a firmer ground in the history of mathematics, recorded in documents that have been preserved.

Keywords: History, Construction, Knowledge, Mathematics.

INTRODUCTION

> "Mathematics is not limited to a system of rules and rigid truths, but it is something human and engaging." (BARONI, 2004).

It is in common agreement among a significant part of researchers in Mathematics Education that History is an important ally in contextualization, in identifying the essence of objects, in the meaningful learning of contents. Mathematics has its origins in the investigation of great thinkers, but also in the development of philosophical thought. In order to demonstrate these observations, we will briefly review some of the main contributions of researchers, both in the area of investigation and in the evolution of logical-philosophical reasoning, within the context of the development of Mathematics as a Modern Science.

After significant contributions due to the Babylonians and the beginnings of positional notation (cuneiform numeration - around 2000 BC), mathematics begins to organize itself as knowledge in Thales of Miletus in approximately 624-548 BC. A man of rare intelligence and considered the first philosopher and founder of Western philosophy in 600 BC. and the first of the Seven Sages. The proofs of some theorems are attributed to him, a fact that earned him the designation of "the first mathematician".

With Pythagoras of Samos (Greek mathematician and philosopher – approximately 570/571-497/496 BC) came the foundation of the Pythagorean school, a secret society whose most notable feature was the moral basis of conduct. He is supposed to have created the words "philosophy" (love of wisdom) and "mathematics" (what is learned). Considered the father of mathematics, his contributions show little intellectual structure and, perhaps, no characteristic of philosophical discussion of principles. However, it is certain that the Pythagoreans related mathematics more to the love of wisdom than to the demands of practical life. The Pentagram especially symbolized the Pythagorean school, whose horizons advanced into mysticism, arithmetic, logistics, cosmology, figurative numbers, numeration, proportions.

Zeno of Elea – the Eleatic – Greek philosopher who lived from 490 to 430 BC, whose method "was dialectical, anticipating Socrates in this indirect mode of argument: starting from the premises of his opponents, he reduced them to absurdity" (BOYER, 1996, p. 51), the trivium of grammar, rhetoric and dialectics (logic) is attributed, which together with the quadrivium (arithmetic, music, geometry, astronomy) of Archytas constituted the seven liberal arts. He is well remembered for his paradoxes, whose aim was, for the most part, to overturn the Pythagorean thesis that "space and time can be thought of as consisting of points and instants" (BOYER, 1996, p. 51). However, space and time have an intuitive property known as continuity and this is the fulcrum of their reasoning.

Some anthropologists claim that, in the Paleolithic era, the man who lived in the region of Europe already knew the symbolism of the numbers 3 (masculine) and 4 (feminine). In the Middle Ages, associations: △ = ♂ = 3 e □ = ♀ = 4, reappear in Europe. In Spagyria, whose sources are diverse, the symbol of perfection is in the junction of a square and a triangle, something that greatly influenced the art of this period. Spiritual (three) and material (four) symbolism illustrate the hermeneutics of the time, and serve as a basis for the distinction between the liberal arts (the trivium and quadrivium) of medieval universities.

Plato, a Greek philosopher who lived from 427 to 347 B.C., did not contribute with noteworthy technical mathematical results, but was considered a "maker of mathematicians". Converted by Archytas (a Pythagorean, with whom he was a friend, who had established a government in Tarentum whose principles were based on philosophy) to a mathematical vision, "Plato put his ideas about regular solids in a diary entitled Timaeus, presumably named after a Pythagorean, who serves as the main interlocutor" (BOYER, 1996, p. 58). Often called "cosmic bodies" or Platonic solids, regular polyhedra are so recognized because of the way he applied them to the explanation of scientific phenomena.

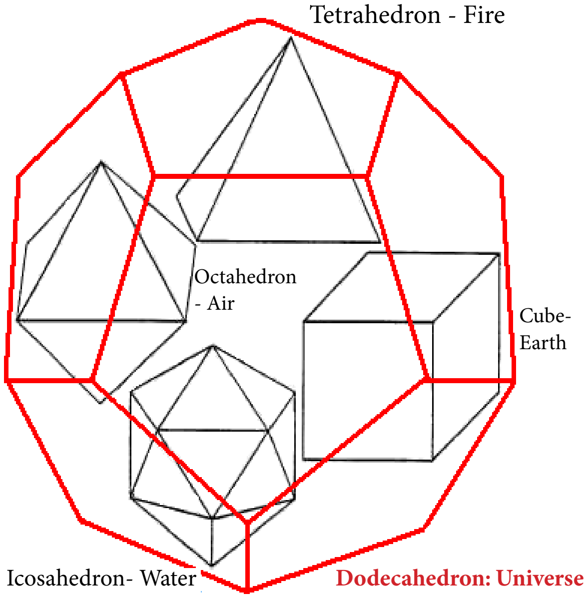

Figure 1 - Plato's solids.

Source: Personal archive.

In this period, the connection between Mathematics, Astronomy and Religion, interconnected as the "representation of the divine", was quite expressive. The preoccupation with knowing where we came from and where we are going already dominated the main researches of these men of science.

Euclid of Alexandria (Greek mathematician - 3rd century BC) was one of the first-rate scholars of the time, called to the position of professor at the Museum - a school created by Ptolemy I in Alexandria in 306 BC, after the end of the Hellenic period. in 323 BC, with

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the death of Alexander the Great. Author of The Elements, a work considered the oldest to reach us, comprising thirteen books.

Archimedes (Syracuse, present-day Sicily – 287-212 BC – Greek inventor) was the greatest mathematician of all antiquity. Among his many works, we can highlight the General Law on the Lever (give me a lever long enough and a fulcrum to support it and I will move the Earth), the Hydrostatic Principle and the Measure of the Circle, with results very close to the value of "π" that we know approximately today.

Hero of Alexandria, Greek mathematician and mechanic (AD 65-125), is remembered in the History of Mathematics, especially for the formula for calculating the area of any triangle, given the measures of its sides. He is remembered in the history of science as the inventor of the first prototype steam engine and thermometer. He studied the phenomena of light reflection, both in convex or concave mirrors and in plane mirrors, and wrote treatises on mathematics (measurements of areas and volumes), in addition to numerous treatises devoted to mechanics.

With these brilliant scholars begins the formalization of knowledge, creative genius, contextualization and dialectical expressiveness.

WHAT IS CURIOUS ABOUT THE DEVELOPMENT OF MATHEMATICS

Observing the contributions of each of these remarkable philosophers and/ or mathematicians, we began to outline the curiosity and incredible visionary notion that each of them developed, even if unintentionally, when envisioning ideas and thought routines that would survive to this day. Let's see some of these developed ideas that, in a way, constitute some algorithms and properties that we use nowadays to perform certain numerical calculations.

In a very similar way to the model used today, addition and multiplication were carried out in India, except for minor variations. A curious form of multiplication used by them has survived to us, recognized by several names: lattice multiplication, lattice multiplication ("the current word jalousie seems to come from Italian lattice, and means venetian in France, Germany, Holland and Russia" (BOYER), 1996, p. 148)), cell, grid or quadrilateral multiplication. Here's an example and step-by-step how to use this method:

a) The tables from one to nine are written in vertical strips, where the ten and one digits are separated by a diagonal, as shown in figure 2a;

b) In our example, the operation to be performed will be: 8989. 9975; thus, from the vertical strips, we will use two corresponding to eight and two corresponding to nine, forming the number 8989, as shown in figure 2b;

c) Observing the horizontal lines, we will concentrate on the lines corresponding to nine (twice), seven and five, in that order, forming the number 9975, as shown in Figure 2c;

d) The final result will be the addition of the values located on the same diagonal, from right to left, remembering that the decimal digit, if any, will participate in the addition of the next diagonal, as shown in figure 2d.

During the caliphate of al-Mamum (AD 809-833), a "House of Wisdom" comparable to the ancient Museum of Alexandria was established in Baghdad. Among the masters who passed through there was a mathematician and astronomer named Mohammed ibu-Musa al-Khowarizmi (780-850 AD). Among his works is ''hindorum '' (On the Hindu Art of Calculation), where he gives such a thorough exposition of Hindu numbers that today our numerical system is said to be Hindu-Arabic (sometimes considered more Arabic than

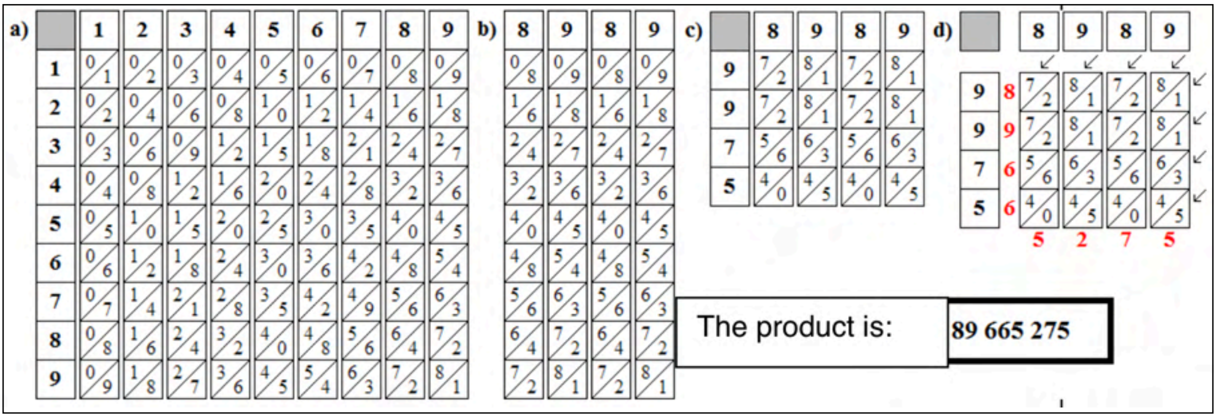

Figure 2 - Multiplication in lattice.

Source: Personal archive.

Hindu, the which is a false thought). His most important book - Al-jabr Wa'l muqabalah - has a study where the possibilities regarding linear and quadratic equations are exhausted, through a systematic and very didactic exposition (a fact that earned him recognition as the "father of algebra").

In the period from 965 to 1039, in Basra, now Iraq and the city of Cairo, lived an Arab physicist and mathematician, whose name was ibn-al-Haitham, known in the West as Alhazen. He wrote a treatise entitled Treasure of Optics where he writes about "the structure of the eye, the apparent increase in the size of the Moon when it is near the horizon, and an assessment of the height of the atmosphere, assuming that twilight lasts until the Sun reaches 19° below the horizon." horizon" (BOYER, 1996, p. 164). Due to this treaty, we know beautiful works of art that have the Theory of Perspective as their basis. Engineering, architecture, proportionality, the Renaissance, all thank these brilliant ideas that contemplate this theory. The sensation of depth, the proportionality between the elements, the mosaic of lights and colors, the dynamism, represent, with more expressiveness, nature, the real.

Figure 3 – "As Meninas" de Diego Velázquez (perspective and movement).

Source: Available at: https://sapienciacultural.wordpress.com/2015/01/20/as-meninas-diego-velazquez-a-infanta-nao-e-a-protagonista/. Acesso em: 24 ago. 2016.

In the age of Modern Philosophy lived the most important Swiss mathematician of all time. His name was Leonhard Euler (Basel, 1707 – St. Petersburg, 1783). A researcher with an international reputation and fantastic production, Euler was not shaken even by the blindness that accompanied him for the last seventeen years of his life. His mathematical notation, his fundamentals of analysis, his works with infinite series, identities, differential equations, probability, number theory and spatial analytic geometry, are some of his notable contributions. Euler is credited with initiating studies in the area of graph theory, a fact well illustrated by the famous problem of the seven bridges of Königsberg.

The city consisted of two "coastlines" A and B, two islands C and D, and seven bridges as shown in Figure 4 (I). The problem: starting from any point (coast or island) is it possible to walk once on all the bridges and return to the starting point? First, Euler transformed each starting location into a point, calling them vertices, odd or even, according to the number of edges (bridges) connected to them, as shown in figure 4 (II). Considering this situation, Euler made three observations:

1. If the bridge connecting the two islands were excluded, the problem would have "intermediate solutions", due to the fact that we have even and two odd vertices, as shown in Figure 4 (III). We can start from an odd vertex, pass through all the bridges only once and arrive at the other odd vertex;

2. If there was a bridge connecting the two coastlines, the problem would have a solution according to the original problem, as all vertices are even – Figure 4 (IV);

Thus, he concluded that the original problem had no solution because all vertices are odd, as shown in figure 4 (V).

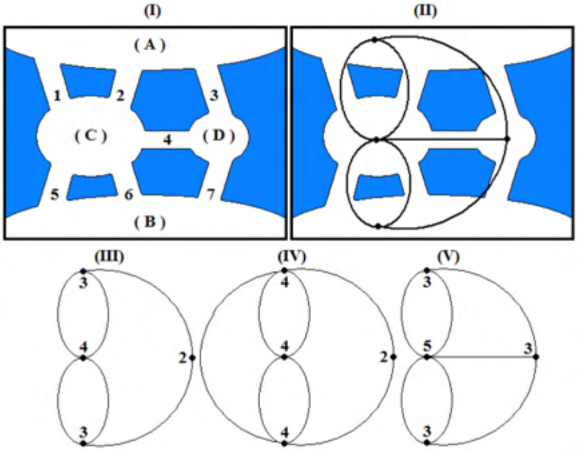

Figure 4 – The problem of the seven bridges of Königsberg.

Source: Personal collection.

After these considerations, we see that one of the most suggestive ways of teaching is to lead the student to reflection based on a story, a case, a fable, a metaphor, a historical problem. Often these facts have the property of sharpening the memory and, through it, creating links with significant learning, and, therefore, the person remembers the story and ends up associating it with the reflective fact that was attached to it. Therefore, considering that talking about any content, whatever the subject, using, from time to time and with an extremely accentuated criterion of selection, of stories, a very important teaching process is developed for the memory and for the own conscience and meaningful learning. Mathematics topics are no exceptions. These same arguments can also be used and the story becomes a vehicle for a true feeling of appropriation of knowledge and the awakening of reflection.

FINAL CONSIDERATIONS

Our entire tour of this small part of the History of Mathematics reveals steps in the development of mathematical reasoning and, consequently, in the establishment of guidelines to guide science in general. We have seen that it is fundamental for medieval mathematics to be taught in institutions to allow a better understanding of Sciences and Philosophies. In the flow of time, it was not by chance that scientific development lived (and still lives) on paradoxes, on struggles between opposites. Technological advances bring progress, but they also bring discord. We live on the edge between love and hate, war and peace, chaos and logic.

> *A man's Wisdom is measured through his usual ability to "simplify" knowledge, so that it is real, be tasted by the one who learns, by the human being who, in a "future present", "will", also, teach.* [...] It is necessary to make "things" simple and not to "make simplisms". Mathematics as a phenomenon

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> is basic material for the elaboration of a more meaningful and less obscure world. Meaning that is shown through a meaning, from the moment it becomes necessary.
>
> It is necessary to "dry up" the sea of lack of meanings, of senses, which has flooded classrooms through formulas and equations. Teacher training must be improved with knowledge of the epistemology of Mathematics, with a greater understanding of the structure of sciences as well as the space they occupy in the intellectual system. To this end, the history, philosophy and sociology of science can contribute to a more comprehensive understanding of scientific matters, that is, they can contribute to overcoming these problems. (ARAÚJO, 2013, p. 118-119).

When we say: everything, before, always, so much, greater, more, each, moment, end, when, one, many, late, too much, early, never, eternal, we have the idea of quantity, of time, of data for the knowledge. The word convinces because it makes sense. It makes sense because it is also positional. Dialectics is its maximum moment of glory or its deepest defeat. As Plato visualized, the word is pharmakon – cure, poison, numb. Our History goes further when we think of other origins, in other words. For example, say: radix quadratum 81 aequalis 9 – from Latin: the side (radix) of the square (quadratum) 81 is equal (aequalis) to 9. Therefore, we say: square root of eighty-one is equal to nine ( ), where the semiotic genesis of the root symbol ( ) is in the letter "r" of radix. Similarly, we have: quadractus radix 9 aequalis 81 – is 81 the area of the square of side 9. Therefore, we say: Squared of 9 or 9 squared is equal to 81 (9). To conclude: cubus radix 3 aequalis 27 – the volume of the cube of side 3 is 27. Therefore, we say: the cube of 3 or 3 cubed is equal to 27. These brief observations tell a little about the history of the signa notae (evolution of symbols).

In the midst of all this cultural root, education tries to survive and fulfill the fundamental role based on the development of people and societies. She points to the need to build a school aimed at training citizens, who live "in an era marked by competition and excellence, in which scientific progress and technological advances define new requirements for young people who will enter the world of work" (BRAZIL, 1998, p. 05). However, these "new technologies", alone, do not give any meaning to the sciences. Together with History, with Materiality, with Contextualization, we can reach the core of the object studied and understand it better. Such a demand imposes a review of the curricula and methodologies, which guide the daily work carried out by us educators, specialists and researchers in education in our country who, in addition to all these responsibilities, carry on their shoulders the noble role of trainers of new teachers.

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"FIGURA 3" – **As Meninas** de Diego Velázquez (perspectiva e movimento). Disponível em: <https://sapienciacultural.wordpress.com/2015/01/20/as-meninas-diego-velazquez-a-infanta-nao-e-a-protagonista/>. Acesso em: 24 ago. 2016.